A rather remarkable result of Oxtoby and Ulam (and von Neumann) in 1941 is their Homeomorphic Measures Theorem which asserts the following: Let $\mu$ be a Borel probability measure on the $n$-dimensional cube $I^n$ and let $\lambda_n$ be $n$-dimensional volume (Lebesgue) measure. If $\mu$ is positive on open sets, and zero for points and boundary, then there is a homeomorphism $h$ of the cube to itself, so that $\mu(E) = \lambda_n(hE)$ for all Borel sets $E$. For $n = 1$, the result is trivial, but when $n \geq 2$ complications of the general case arise, as subsets of dimension strictly less than $n$ can support some or all of the mass for $\mu$. The Homeomorphic Measures Theorem is a basic tool in the study of volume preserving homeomorphisms.

In this talk I consider the question of choosing the “transfer” homeomorphism $h$ (so $\mu = \lambda_n h$) so that it depends continuously on the measure $\mu$, when the space of measures has the weak topology and the homeomorphism group has the topology of uniform convergence. We are unable to prove the result in full generality. We extend some results of Fathi, and also Peck to the infinite dimensional setting of the Hilbert cube $I^\infty$. This is joint work with V. Peck (IBM) and the talk should be accessible to all attending.