TRAINING MIDTERM 1

The actual midterm will probably be a little bit shorter.

1.– On the set $\{T, F\}$, we consider three binary laws:

(i) AND is defined by: $x \,\mathrm{AND}\, y = T$ if and only if $x = y = T$.

(ii) OR is defined by: $x \,\mathrm{OR}\, y = T$ if and only if at least one of x or y is T.

(iii) OR is defined by $x \,\mathrm{OR}\, y = T$ if and only one exactly one among x, y is T.

a.– Which of those laws are group laws?

b.– Among those laws, are there two that are isomorphic to each other? If yes tell which ones, and describe an isomorphism.

2.– Let $\tau \in S_n$ and assume that $\tau^2 = \mathrm{Id}$. Show that all orbits of τ have cardinality either 1 or 2.

3.– Let A be the set of elements of $\mathbb{Z}$ of the form $66a+12b-9c$ for $a, b, c \in \mathbb{Z}$. Show that A is a subgroup of $\mathbb{Z}$. Is A cyclic? If yes, give a generator.

4.– Let A be the set of elemnets in $\mathbb{R}$ of the form $a + b\sqrt{2}$. Show that A is a subgroup of $(\mathbb{R}, +)$. Is A cyclic? If yes, give a generator.

5.– How many elements does have the subgroup of $(\mathbb{C}^*, \times)$ generated by i? Is it cyclic?

6.– Let $G = \mathbb{Z}_{12}$ and $S = \bar{3}, \bar{7}$. Draw the Cayley graph of (G, S). Is S a generating subset of G ?

7.– Show that there three are elements in A_4 that are the product of two disjoint transpositions. Show that with the identity, they form a subgroup of A_4 with four elements. Is this subgroup cyclic?

8.– Let $\sigma \in S_n$ and $(a_1, \ldots, a_m)$ a cycle of S_n. Is $\sigma(a_1, \ldots, a_m)\sigma^{-1}$ a cycle of S_m? if yes, what is its order? what is its support?

9.– Compute $(ijk)(kls)$ where i, j, k, l, s are distinct. Is that a cycle? Otherwise, what is its decomposition as a product if disjoint cycles?