

## TRAINING EXERCISES FOR MIDTERM 2

- 1.– Let  $f : G \rightarrow G'$  be a morphism of groups, where  $G$  is a finite group. Show that  $|f(G)|$  divides  $|G|$ .
- 2.– Let  $f : G \rightarrow G'$  be a morphism of group. Let  $H$  be a normal subgroup of  $G'$ . Show that  $f^{-1}(H)$  is a normal subgroup of  $G$ .
- 3.– Let  $G = \mathbb{R}/\mathbb{Z}$ . Show that  $G$  has elements of any order  $n \geq 1$ .
- 4.– Let  $T$  be the set of upper triangular  $2 \times 2$ -matrices  $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$  with  $a, b, c \in \mathbb{R}$ ,  $a \neq 0$ ,  $c \neq 0$ .
  - a.– Show that  $T$  is a group for the multiplication of matrices.
  - b.– What is the center  $Z$  of  $T$ ?
  - c.– What is the commutator group  $C$  of  $T$ ?
  - d.– Can you describe the group  $T/C$ ?
- 5.– What is  $(Z \times \mathbb{Z})/\langle(2, 2)\rangle$ ?
- 6.– Let  $G$  be a group,  $H$  a normal subgroup of  $G$  of index  $n$ . Show that for all  $x \in G$ , we have  $x^n \in H$ .