Malthus, the Industrial Revolution, and the Technology Multiplier

Gary H. Jefferson
Department of Economics and the International Business School
Brandeis University
Jefferson@brandeis.edu

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1. Introduction

The conventional interpretation of the industrial revolution is that the acceleration of living standards that marked the revolution was driven by a more rapid pace of technological progress. The acceleration of technological change, occasioned by the scientific revolution, was characterized by an ability to devise new industrial methods faster than was previously possible.

This paper argues that the change that marked the industrial revolution was even more fundamental; it was a qualitative change in which humankind learned how to devise physical goods that embodied new techniques. The industrial revolution introduced a whole new factor of production that had not previously existed. The essential change was the ability to invent and embody a wide range of new technologies into physical capital.

We demonstrate this change by comparing two version of Solow's neoclassical growth model. The first, characterized as the Malthusian-Solow model, includes only land and labor as its factors of production. The second model – the modern Solow model – substitutes capital for land. We investigate the implications of the widespread use of this new factor of production.

2. The pre-industrial Malthusian era

In order to demonstrate the role of technical change in the pre-industrial Malthusian era, we describe a Malthusian world in which there exists (i) two factors – land and labor, (ii) diminishing returns to land, the fixed factor, and (iii) passion of the sexes, i.e. a tendency for men and women to use whatever surplus they generate to support procreation. These, in addition to the assumption of sporadic technical change, are the conventional assumptions of the Malthusian economic system. We model this world using a modified version of the standard Solow neoclassical growth model with labor (L) and land (T) (rather than capital) as the two factors.

The production technology is characterized by Q = Af(T,L), which with constant returns to scale can be represented in the following intensive or percapita form: q = Af(l), where q is yield and l is the labor-land ratio.

In the Malthusian steady state, the birth rate (β) and the death rate (δ) are equal, i.e. $\beta = \delta$, so that population growth is zero at the subsistence wage. The subsistence wage, w*, is just sufficient to sustain the steady state labor-land ratio, l*.

In the Malthusian world, let α represent the cost of raising a child, while s is the share of income generated per unit of land allocated to child rearing. The number of surviving children per household is therefore sq/α. Because in the steady state, the number of births and deaths are equal, the Malthusian Solow steady state condition is:

$$sq/\alpha = \beta l = \delta l.$$

The steady state condition is shown in Figure 1. At l_0*, the steady state condition holds, i.e. $sq = \alpha\delta l$; both the labor-land ratio (L/T) and the yield (Q/T) are in steady state, and therefore so are productivity and living standards [i.e. Q/T and /L/T].

How does technical change enter the model? Malthus' story about technical change was that (i) it was limited, hence living standards never rose by much, and (ii) whenever technical change did occur, so that household income rose above subsistence levels to allow for discretionary time and income, men and women would produce more children so as to absorb the surplus and drive living standards back down to subsistence levels.

We demonstrate this economic story in Figure 1. Technical change shifts the productivity parameter from A_0 to A_1 thus shifting up the production function. Higher productivity raises incomes above the subsistence level. The creation of discretionary time and income results in the production of more children, which in turn causes the labor-land ratio to rise to l_1*. The combined rise in yield and the labor-land ratio drives productivity and living standards back to their subsistence levels.

We see in the model, therefore, that unlike in the conventional Solow model, technical change exerts only a temporary increase in living standards in the Malthusian two-factor world. Moreover, no amount of technical change will create higher sustained living standards. Above-subsistence levels of discretionary simply lead to higher rates of human reproduction, which raise labor-land ratios and depress productivity and living standards toward their subsistence levels.

3. Technical change in pre-industrial times

In this section, we review the literature on the industrial revolution. We attempt to demonstrate that in the pre-industrial era technological progress was either non-existent or was principally of the disembodied variety. Examples of pre-industrial technical change include the introduction of letting fields lie fallow to increase yields or the extension of farming onto new lands. Means for inventing new equipment, seeds, or other inputs to farming that increased productivity were extremely limited. That is, apart from population and labor, the role of factors of production that could be reproduced and used to improve farm productivity were generally unavailable.

4. The introduction of capital goods

Now redirect attention to the modern Solow growth model, which substitutes capital for land as labor's principal complementary factor. An increase in living standards that arises from productivity growth raises savings, as in the Malthus model. Unlike the Malthus model, in the Solow model new surpluses are invested in the accumulation of capital, not in additional children. Capital creates an alternative vent for surplus. Surplus resources, i.e. w-w*, are no longer invested in reproducing children; they are invested in reproducing capital.

5. The technology multiplier

Now let's look at a dynamic feature of the Solow model. Suppose that the modern economy enjoys a one-time, sustained increase in productivity. What impact will it have on output per capita. Using the intensive production function and the steady state equation, we derive the technology multiplier shown in Annex A.

Upon examination of the expression of the technology multiplier, one clear result that we find is that the steady state impact of a rise in productivity varies directly with α, the share of the non-labor reproducible input. As a result, for the Malthusian economy in which $\alpha = 0$, the technology multiplier is just zero. We saw this in Figure 1. With the introduction of capital goods, such that $\alpha > 0$, the technology multiplier, $1/(1 - \alpha)$ becomes greater than zero. Indeed, as $\alpha \rightarrow \infty$, $1/(1-\alpha)$ also$\rightarrow \infty$.

In rate of change terms, it is also the case that an increase in the magnitude of ∞ will increase the dynamic technology multiplier, i.e. $\Delta q/q = [1/(1 - \alpha)]\Delta A/A$.

This analysis leads to a new interpretation of economic progress. Economic progress consists of the on-going expansion of the share of factor returns in total economic output attributable to non-labor reproducible factors of production. While the principal example of this is fixed capital, other examples include education, software, and other forms of reproducible goods and ideas.

6. Sources of expanding reproducibility

Inventions add to the stock of reproducible resources. When inventions, such as synthetic rubber or solar energy, substitute directly for fixed resources, such as rubber trees or fossil fuel, they increase the overall share of production that originates from reproducible goods. By vastly expanding agricultural productivity, the application of hybrid seeds, new fertilizers and pesticides, and controlled irrigation associated with the Green Revolution effectively expanded available land. Labor-saving inventions relax the constraint of the 24-hour day, thereby effectively reproducing time. As the share of reproducible goods increases, the constraint of diminishing returns to investment, broadly defined, is relaxed. As shown by the technology multiplier, as the magnitude of α grows, the effect of productivity gains on living standards increases.

7. Estimates of capital's share

Assuming international capital mobility and capital output ratios rising with development, then rK/Q, i.e. capital's factor income share should rise over time. Hence the technology multiplier rises over time. We test this using World Bank data.

7. Conclusion

The essential distinction between the pre-industrial era and the modern era is the introduction of capital, particularly successively new vintages of capital, which substitute for labor as the principal "vent for surplus" that arises from technological progress.

This view of technological progress and economic change has a broad set of implications. These include:

- The so-called "New Economy" can be interpreted as an increase in α resulting from the invention and introduction of information technologies.

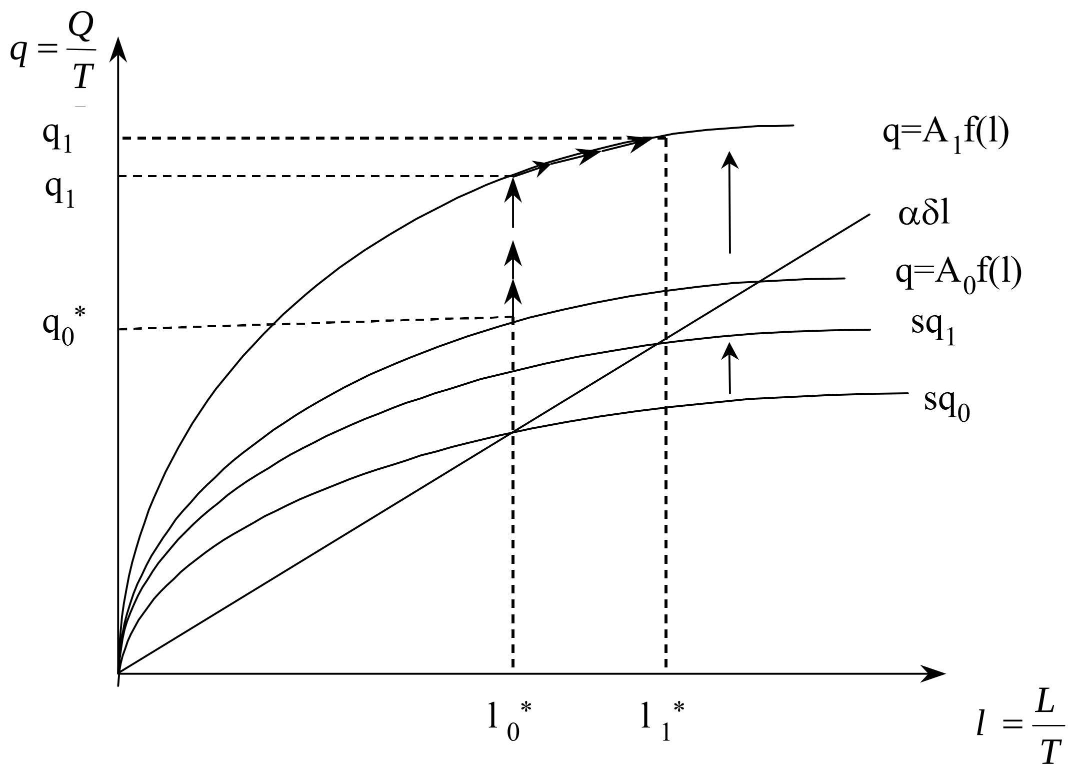
- The model and its results underscore the importance of the economic theory of fertility, i.e. the choice between investing in children and in other goods.

- The tendency for rich countries having larger α's than poor countries causes the same magnitude of technological progress to have less impact on living standards in poor countries than in rich countries.

- The productivity of unaugmented – i.e., low-skill – labor increases as it enjoys more complementary inputs. This explains why unskilled labor that moves from a poor country to a rich country, it is able to produce more.

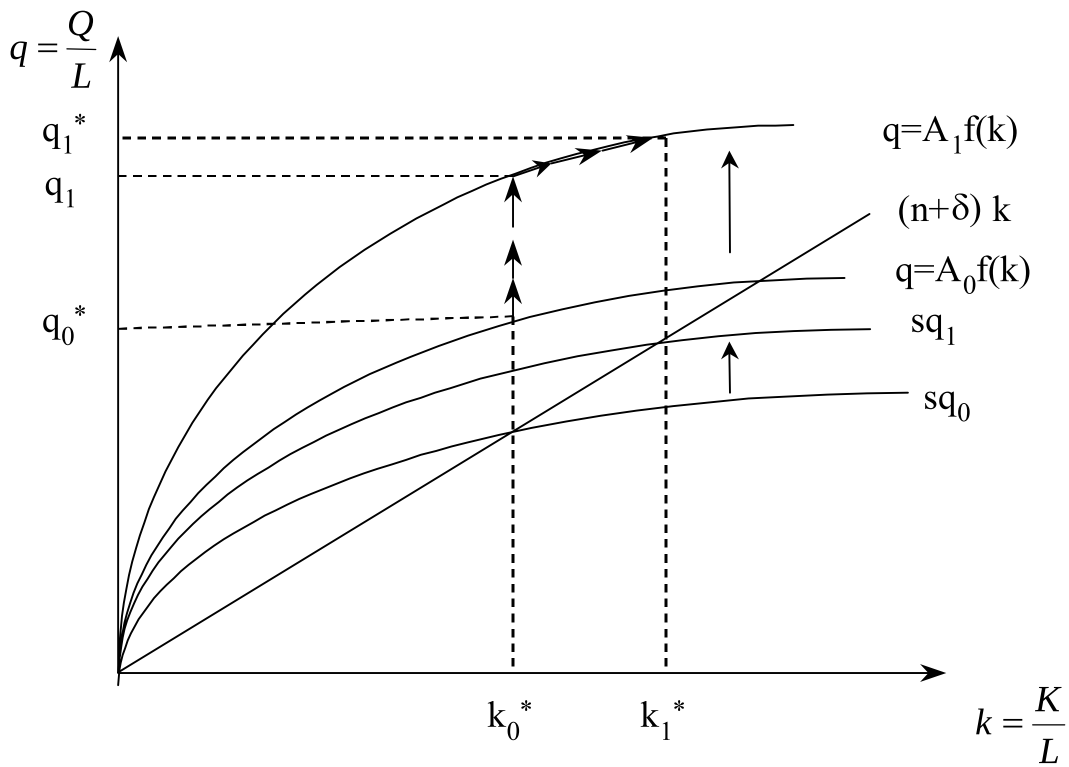
- However, an increase in α can lead to further income inequality, as reproducible substitutes are discovered and deployed for physical labor. The relative return to unaugmented labor declines.

Figure 1
Malthusian-Solow Growth Model



s-s condition: $sq=\alpha\delta l$

Figure 2
Solow's Model of Modern Economic Growth



s-s condition: sq=(n+ δ)k

Annex 1: The Technology Multiplier

the production function: $q = Ak^{\alpha}$

totally differentiate: $dq = dAk^{\alpha} + \alpha Ak^{\alpha-1}dk$

divided by dA and multiplied by A/q: $q_A = 1 + \alpha k_A$, (1a)

where $q_A = (dq/dA)(Q/q)$ and $k_A = (dk/dA)(A/k)$.

Equation (1a) shows that $q_A > 1$.

What is k_A?:

steady-state condition: $sq = (n+\delta)k$

solve for k: $k = [s/(n+\delta)]q$ (2a)

which implies $K/Y = s/(n+\delta)$.

Diff. eq. (2a) w.r.t. q: $dk/dq = s/(n+\delta)$. (3a)

Also, $k_A = [(dk/dq)(q/k)](dq/dA)(A/q)]$ (4a)

Subst. (3a) into (4a): $k_A = [(s/(n+\delta)(q/k)]q_A$. (5a)

Subst. (5a) into (1): $q_A = 1/\{1-\alpha[sq/(n+\delta)k]\}$ (6a)

In the S-S, $sq = (n+\delta)k$, so (6a) simplifies to:

$$\mathbf{q_A = 1/(1-\alpha)}.$$

Figure 3
Technology Multiplier (q_A)
$q_A=1/(1-\alpha)$