The Geometric Satake Equivalence for Dummies (and for Quantum Dummies)

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The Geometric Satake Equivalence is an equivalence of categories between the representations of a semisimple Lie algebra $\mathfrak{g}$ and some geometrically defined category (of perverse sheaves). These are both difficult categories to get a handle on, and the usual proofs of this equivalence are far from explicit. Thankfully, both these nasty categories have subcategories which can be made explicit and combinatorial in a nice way (in type A). In other words, the morphism algebras of these subcategories can be given simply by generators and relations. This makes it easy to give a straightforward proof of the equivalence of these subcategories, and hence of the original categories. If you ever wanted to get your hands dirty with Geometric Satake, this is a useful tool.

Moreover, the morphism algebras above admit $q$-deformations, leading to a Quantum Algebraic Satake Equivalence (with no geometry, at the moment). What is Satake-equivalent to representations of quantum $\mathfrak{sl}_2$ at a root of unity? Come and find out.