The generalized Sato-Tate conjecture

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The Sato-Tate conjecture is concerned with the statistical distribution of the number of points on the reduction modulo primes of a fixed elliptic curve defined over the rational numbers. It predicts that this distribution can be explained in terms of a random matrix model, using the Haar measure on the special unitary group SU(2). Thanks to recent work by Richard Taylor and others, this conjecture is now a theorem.

The Sato-Tate conjecture generalizes naturally to abelian varieties of dimension $g$, where it associates to each such abelian variety a compact subgroup of the unitary symplectic group USp$(2g)$, the Sato-Tate group, whose Haar measure governs the distribution of certain arithmetic data attached to the abelian variety. While the Sato-Tate conjecture remains open for all $g > 1$, I will present recent work that has culminated in a complete classification of the Sato-Tate groups that can arise when $g = 2$. I will also describe extensive numerical computations that support the conjecture, along with animated visualizations of this data.

This is joint work with Francesc Fité, Victor Rotger, and Kiran S. Kedlaya.