Math 20a Quiz 2 March 6, 2012

**Instructions:** Read the problems carefully before you start thinking or writing. No calculators allowed; leave any quantities such as $0.4e^5$ without evaluating numerically. There are a total of 20 points.

1. [12 points] (a) Find an equation of the tangent plane to the graph of the function $z = f(x, y) = e^{xy^2}$ at the point $P(0.5, 2, e^2)$.
   (b) Find the linearization $L$ of $f$ at the point $P$.
   (c) Use your answer to (b) to estimate $f(0.4, 2.05)$.

   **Answer:** (a) $f_x = y^2e^{xy^2}$ and $f_y = 2xye^{xy^2}$; thus $f_x(0.5, 2) = 4e^2$ and $f_y(0.5, 2) = 2e^2$. The equation of the plane is given by
   $$z = f(0.5, 2) + f_x(0.5, 2)(x - .5) + f_y(0.5, 2)(y - 2)$$
   or
   $$z = e^2 + 4e^2(x - .5) + 2e^2(y - 2) \iff z = e^2(4x + 2y - 5).$$
   (b) The right hand side of the equation of the plane is exactly the linearization of $f$ at the given point:
   $$L(x, y) = e^2 + 4e^2(x - .5) + 2e^2(y - 2) = e^2(4x + 2y - 5).$$
   (c) The question is asking for
   $$L(0.4, 2.05) = e^2 + 4e^2(-.1) + 2e^2(.05) = e^2(1 -.4 + .1) = .7e^2.$$ Note that may people wrote instead that $f(0.4, 2.05)$ is equal to the right hand side of the above formula, which is not true – the equality is only approximate. I was taking off one point for that, just as a formal reminder.

2. [8 points] Evaluate the following limit, or show that it does not exist:

   $$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$$

   **Answer:** The limit does not exist. If you set $y = 0$ (thus calculating the limit along the $x$-axis), then the limit becomes
   $$\lim_{x\to0} \frac{x^2}{x^2} = \lim_{x\to0} 1 = 1.$$ On the other hand, if you set $x = 0$, the function becomes 0, so the limit along the $y$-axis is 0. Since these limits do not agree, the limit does not exist.