# Math 21b Review Sheet for the Final Exam

Everything written in the midterm review sheet is still valid. Here is what **additional exam problems** may look like.

• Find the largest and smallest values, and points where these values are attained, of a function $f : D \to \mathbb{R}$, where $D$ is a subset of $\mathbb{R}^n$. (This may involve: locating critical points of $f$ in the interior of $D$, and solving a Lagrange multiplier system for the boundary of $D$. A problem of this type may happen to be a word problem, in which case you will need to translate it into a mathematical language.)

• Find all the critical points of a function $f : \mathbb{R}^n \to \mathbb{R}$, and, for $n = 2$, classify them into local maxima, local minima or saddles. (If the second derivative test gives no information, you may still be asked to try to reach a conclusion.)

• Compute $\int_D f(\mathbf{x})\, d\mathbf{x}$, where $D$ is a subset of $\mathbb{R}^n$, by reducing it to an iterated integral.

• Conversely, given an iterated integral, write it as an integral over some $D$, and perhaps use it to compute the integral, by choosing a different order of integration.

• Use integration to compute the area (volume) of a region in $\mathbb{R}^2$ ($\mathbb{R}^3$), and also its mass if a density is given.

• Represent regions $D$ in $\mathbb{R}^2$ ($\mathbb{R}^3$) in polar (cylindrical, spherical) coordinates, and use it to compute integrals of functions over $D$. Do the same for some other changes of variables, computing Jacobians of the corresponding transformations (see Theorem 4.4 and Example 6 in Chapter 7, pp. 342–343).

• Compute the integral of a vector field $\mathbf{F}$ over a curve $\gamma$ in $\mathbb{R}^n$: either using the definition (that is, parametrizing $\gamma$ and calculating expression (1.1) on p. 368), or using Theorem 1.3 in case when $\mathbf{F}$ is in the form $\nabla f$, or choosing a more convenient curve in case when $\mathbf{F}$ is conservative but its potential is not given (see Example 2 on p. 414).

• Use Green's Theorem (Theorem 1.2 on p. 400) to compute line integrals (see Problems 1–4 on p. 408) or double integrals (see Problem 11 on p. 408).

• Given a vector field $\mathbf{F}$, decide whether or not it is conservative (Theorem 2.6 on p. 414), and if it is, find its potential, either by definite integration as in Theorem 2.1, pp. 410, or by indefinite integration as in Examples 5 and 6, pp. 417–418.

The complete **list of sections** covered since the midterm is:
6.4 (except for F), 6.5, 7.1, 7.2, 7.3 (except for Theorem 3.7), 7.4, 8.1, 9.1, 9.2.

There will be no problems explicitly asking you to make a **sketch** (for example, of a region of integration), yet it may happen to be helpful for the solution.

## Good luck!