# Math 22a Fall 2009 Homework 2 Answers and solutions

14. $[-1, 3, 4]\cdot[x, -3, 5] = -x - 9 + 20 = 0, \ -x + 11 = 0, \ x = 11$
(6)

22. $\cos\theta = \dfrac{[1, -1, 2, 3, 0, 4]\cdot[7, 0, 1, 3, 2, 4]}{\sqrt{1+1+4+9+0+16}\sqrt{49+0+1+9+4+16}}$
(4)

$= \dfrac{34}{\sqrt{2449}} \approx 0.687044$, so $\theta = \cos^{-1}(\dfrac{34}{\sqrt{2449}}) \approx 46.6°$.

23. $[4, 1, -1] - [2, 0, 4] = [2, 1, -5]$,
(6) $[6, 7, 7] - [2, 0, 4] = [4, 7, 3]$,
$[2, 1, -5]\cdot[4, 7, 3] = 8 + 7 - 15 = 0$. Since the vectors are perpendicular, the triangle formed by the three points is a right triangle.

43. $$(\mathbf{v} - \mathbf{w})\cdot(\mathbf{v} + \mathbf{w}) = \mathbf{v}\cdot\mathbf{v} - \mathbf{w}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{w} - \mathbf{w}\cdot\mathbf{w}$$
(10) $$= \mathbf{v}\cdot\mathbf{v} - \mathbf{w}\cdot\mathbf{w}$$
$$= \|\mathbf{v}\|^2 - \|\mathbf{w}\|^2.$$

Thus $\mathbf{v} - \mathbf{w}$ and $\mathbf{v} + \mathbf{w}$ are perpendicular if and only if $(\mathbf{v} - \mathbf{w})\cdot(\mathbf{v} + \mathbf{w}) = 0$, which occurs if and only if $\|\mathbf{v}\|^2 - \|\mathbf{w}\|^2 = 0$, which happens if and only if $\|\mathbf{v}\| = \|\mathbf{w}\|$.

§1.3

12. $AC = \begin{bmatrix} -13 & 14 \\ 11 & -6 \end{bmatrix}$; $(AC)^2 = \begin{bmatrix} 323 & -266 \\ -209 & 190 \end{bmatrix}$
(3)

13. $(2A - B)D = \begin{bmatrix} -8 & 1 & 8 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 3 & 5 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 27 & -35 \\ -4 & 26 \end{bmatrix}$
(3)

18. a) $A^2 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$
(2)

b) $A^7 = (A^2)^3A = \begin{bmatrix} -8 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & -8 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 128 & 0 \\ -16 & 0 & 0 \end{bmatrix}$
(4)

32. The $(i,j)$th entry of $(AB)^T$ is the $(j,i)$th entry in $AB$, which is
(0)

$(j$th row of $A)\cdot(i$th column of $B)$
$= (i$th column of $B)\cdot(j$th row of $A)$ (Theorem 1.3)
$= (i$th row of $B^T)\cdot(j$th column of $A^T)$,

which is the $(i,j)$th entry of $B^TA^T$.

39. Since $(AA^T)^T = (A^T)^TA^T = AA^T$, we see that $AA^T$ is symmetric.
(6)

43. $(A + B)(A - B) = A^2 - B^2$ if and only if $AB = BA$. Let
(8) $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. Then $AB \neq BA$,

$(A + B)(A - B) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, and

$A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

44. Consider the decomposition $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$. Now
(0) $B = \frac{1}{2}(A + A^T)$ is symmetric. Letting $C = \frac{1}{2}(A - A^T)$, we find that $C^T = \frac{1}{2}(A - A^T)^T = \frac{1}{2}(A^T - (A^T)^T) = \frac{1}{2}(A^T - A) = -C$, so $C$ is skew symmetric. This decomposition $A = B + C$ thus expresses $A$ as the sum of a symmetric matrix and a skew symmetric matrix.

For uniqueness, suppose that $A = B_1 + C_1$ is another such decomposition. Then $A^T = (B_1 + C_1)^T = B_1^T + C_1^T = B_1 - C_1$. It follows that $A + A^T = 2B_1$ so $B_1 = \frac{1}{2}(A + A^T) = B$. Similarly, $A - A^T = 2C_1$ and $C_1 = \frac{1}{2}(A - A^T) = C$.

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