(§1.4) Math 22a Fall 2009 Homework 3 Answers & Solutions

a) (3) $\begin{bmatrix} 0 & 2 & -1 & 3 \\ -1 & 1 & 2 & 0 \\ 1 & 1 & -3 & 3 \\ 1 & 5 & 5 & 9 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & 2 & 0 \\ 0 & 2 & -1 & 3 \\ 0 & 2 & -1 & 3 \\ 0 & 6 & 7 & 9 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & 2 & 0 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & 2 & 0 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) (3) $\sim \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

6. a) (3) $\begin{bmatrix} 0 & 0 & 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & 1 & -1 & 3 \\ 2 & 4 & -1 & 3 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & -1 & 3 & 2 & -1 \\ 0 & 0 & 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & 1 & -1 & 3 \end{bmatrix}$

b) (3) $\sim \begin{bmatrix} 2 & 4 & 0 & 5 & 1 & 3 \\ 0 & 0 & 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & 1 & -1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 & 3 & -6 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 & 3 \end{bmatrix}$

8. (3) $\mathbf{x} = \begin{bmatrix} -7 \\ -5 \\ 2 \end{bmatrix}$

16. (4) $\left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 6 & 3 & -8 & 0 \\ 2 & -1 & 5 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 8 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$$\begin{aligned} 2x + y - 3z &= 0 \\ y - 4z &= 2 \\ z &= 0 \end{aligned} \qquad \begin{aligned} x &= -1 \\ y &= 2 \\ z &= 0 \end{aligned}$$

20. (4) $\left[\begin{array}{cccc|c} 1 & -3 & 2 & -1 & 8 \\ 3 & -7 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -3 & 2 & -1 & 8 \\ 0 & 2 & -6 & 4 & -24 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -3 & 2 & -1 & 8 \\ 0 & 1 & -3 & 2 & -12 \end{array}\right]$

$x_3 = r,\ x_4 = s$

$x_2 - 3x_3 + 2x_4 = -12,\ x_2 = -12 + 3r - 2s$

$x_1 = 3x_2 - 2x_3 + x_4 + 8 = 3(-12 + 3r - 2s) - 2r + s + 8 = -28 + 7r - 5s$

22. (4) $\left[\begin{array}{cc|c} 2 & 8 & 16 \\ 5 & -4 & -8 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 4 & 8 \\ 0 & -24 & -48 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 2 \end{array}\right];\quad \mathbf{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

24. (5) $\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 3 & 6 & -8 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 1 & -5 & -5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & -14 & -13 \\ 0 & 0 & 1 & -5 & -5 \end{array}\right]$

$x_1 = -13 - 2r + 14s,\ x_2 = r,\ x_3 = -5 + 5s,\ x_4 = s$

28. (5) $\left[\begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 1 & -2 & 2 & 4 & -1 \\ 2 & -8 & -1 & 0 & 3 \\ 3 & -9 & 4 & 0 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -3 & 1 & 2 & 2 \\ 0 & 1 & 1 & 2 & -3 \\ 0 & -2 & -3 & -4 & -1 \\ 0 & 0 & 1 & -6 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -3 & 2 & 3 & 2 \\ 0 & 1 & 1 & 2 & -3 \\ 0 & 0 & -1 & 0 & -7 \\ 0 & 0 & 1 & -6 & 1 \end{array}\right] \sim$

$\left[\begin{array}{cccc|c} 1 & -3 & 2 & 3 & 2 \\ 0 & 1 & 1 & 2 & -3 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & -6 & -6 \end{array}\right]$ which shows that $\begin{bmatrix} 2 \\ -1 \\ 3 \\ 7 \end{bmatrix}$ is in the span of

$\begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ -8 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}$.

38. (5) $\left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 3 & 6 & b_2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 0 & 0 & -3b_1 + b_2 \end{array}\right]$. The system is consistent if $-3b_1 + b_2 = 0$. Thus we must have $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s \\ 3s \end{bmatrix}$ where s is any real number.

57. (6) Substituting the points in the equation, we obtain $a + b + c + d = 2$, $a - b + c + d = 6$, $16a - 8b + 4c + d = 38$, $16a + 8b + 4c + d = 6$.

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 1 & 6 \\ 16 & -8 & 4 & 1 & 38 \\ 16 & 8 & 4 & 1 & 6 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 0 & 4 \\ 0 & -24 & -12 & -15 & 6 \\ 0 & -8 & -12 & -15 & -26 \end{array}\right] \sim$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & -12 & -15 & -42 \\ 0 & 0 & -12 & -15 & -42 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/4 & 1/2 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 4 & 5 & 14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$.

$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1/2 + s/4 \\ -2 \\ (14-5s)/4 \\ s \end{bmatrix}$;

let $s = 2$, $\begin{aligned} a &= 1 \\ b &= -2 \\ c &= 1 \\ d &= 2 \end{aligned}$, $y = x^4 - 2x^3 + x^2 + 2$.

(*) 58. (0)

a) If $[A \mid \mathbf{c}] \sim [H \mid \mathbf{d}]$ where H has echelon form, then each column of H must have a pivot so there are at least as many rows as columns in H, that is, $m \geq n$.

b) If $m = n$, then $A \sim I$, the $n \times n$ identity matrix, since $A\mathbf{x} = \mathbf{c}$ has a *unique* solution. Thus $[A \mid \mathbf{b}] \sim [I \mid \mathbf{d}']$ and $A\mathbf{x} = \mathbf{b}$ has the unique solution $\mathbf{x} = \mathbf{d}'$.

c) If $m > n$, then the final row of H in part (a) has all zero entries. Let $\mathbf{e}_n$ be the final column of the $n \times n$ identity matrix. Reversing the elementary row operations that reduce A to H, we see that there exists a vector $\mathbf{b}$ such that $[H \mid \mathbf{e}_n] \sim [A \mid \mathbf{b}]$, and the system $A\mathbf{x} = \mathbf{b}$ is therefore inconsistent.

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