Math 22a Fall 2009 Homework 4 Solutions

§1.5

4. a) $\left[\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 8 & 9 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 7/6 & 1/6 & 0 \\ 0 & -1/3 & -4/3 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & -9/2 & 7/2 \\ 0 & 1 & 4 & -3 \end{array}\right]$,
(3)

$A^{-1} = \begin{bmatrix} -9/2 & 7/2 \\ 4 & -3 \end{bmatrix}$

b) $E_1 = \begin{bmatrix} 1/6 & 0 \\ 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 \\ -8 & 1 \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$, $E_4 = \begin{bmatrix} 1 & -7/6 \\ 0 & 1 \end{bmatrix}$,
(3)

$A = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1/3 \end{bmatrix}\begin{bmatrix} 1 & 7/6 \\ 0 & 1 \end{bmatrix}$

7. a) $\left[\begin{array}{ccc|ccc} 2 & 1 & 4 & 1 & 0 & 0 \\ 3 & 2 & 5 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 1/2 & 2 & 1/2 & 0 & 0 \\ 0 & 1/2 & -1 & -3/2 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \sim$
(3)

$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & -1 & 0 \\ 0 & 1 & -2 & -3 & 2 & 0 \\ 0 & 0 & -1 & -3 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7 & 5 & 3 \\ 0 & 1 & 0 & 3 & -2 & -2 \\ 0 & 0 & 1 & 3 & -2 & -1 \end{array}\right]$, $A^{-1} = \begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix}$

b) $E_1 = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,
(3)

12. $A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ -3 & 5 & 0 & 2 \\ 0 & 1 & 2 & -4 \\ -1 & 2 & 4 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & -1 & 3 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -5 & -4 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 5 & -2 \\ 0 & 0 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -5 & -4 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 5 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
(3)

We see that A is not invertible, so the span of its column vectors is not all of $\mathbb{R}^4$.

14. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$
(3)

16. $C = A^{-1}\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 4 & 4 \\ 12 & 11 \end{bmatrix}$
(4)

17. $C = A^{-1}\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix}A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix}A^{-1} =$
(5)

$\begin{bmatrix} 2 & 6 & 11 \\ -1 & 7 & 10 \\ 11 & 8 & 22 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 46 & 33 & 30 \\ 39 & 29 & 26 \\ 99 & 68 & 63 \end{bmatrix}$

21. Since $\begin{bmatrix} 2 & 4 & 2 \\ 1 & r & 3 \\ 1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & r-2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & r \end{bmatrix}$, the matrix is
(5)
invertible for any value of r except $r = 0$.

22. If A and B are row equivalent, then there is a sequence
(6) $E_1, E_2, \ldots, E_k$ of elementary $m \times m$ matrices such that $B = E_kE_{k-1}\cdots E_2E_1A$. Thus $C = E_kE_{k-1}\cdots E_2E_1$ is the required invertible matrix.
Conversely, if $B = CA$ and C is invertible, then C may be expressed as a product of elementary matrices as shown in the text. Therefore, B and A are row equivalent.

26. Let B be a matrix such that $A^2B = BA^2 = I$. Then
(6) $B(A^2) = (BA)A = I$ so A is invertible and $A^{-1} = AB = BA$.

(6)
29. a) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

b) If $A^r = O$ and A is invertible, then $A = A^r(A^{-1})^{r-1} = O(A^{-1})^{r-1} = O$, that is, $A = O$, which is a contradiction since A is invertible.

30. a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(0)
b) If $AA = A$ and A is invertible, then $A^{-1}(AA) = A^{-1}A$, $(A^{-1}A)A = I$, $IA = I$, so $A = I$.

§1.4

48. $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$
(3)

50. $\begin{bmatrix} 0 & 6 & 0 & 1 \\ -2 & 1 & 0 & 0 \\ 0 & -18 & 1 & -3 \\ 1 & 0 & 0 & 0 \end{bmatrix}$
(3)

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