

## Math 22a Review Sheet for Exam #2

Below you will find the list of what you are supposed to know. References are made to the corresponding pages in the book. The exam problems will be modelled over homeworks and quizzes. It goes without saying that a lot of material from Part I of the course will be used.

### 1. You need to understand what is:

a linear transformation (142) and its standard matrix representation (146), kernel and range of a linear transformation (148);

a vector space (180–181), linear combination of vectors and the span of a set of vectors (191), a subspace of a vector space (192–193), linear dependence and independence (194), basis of a vector space (197), a finitely generated vector space (198), dimension (199), a coordinate vector relative to an ordered basis (205), a linear transformation (213) and its matrix representation relative to two ordered bases (223);

a determinant of a  $2 \times 2$  (239) and  $3 \times 3$  matrix (245), the cross product of two vectors in  $\mathbb{R}^3$  (241), a cofactor of an element of an  $n \times n$  matrix (251), a general definition (252) and properties (256–259) of the determinant, Cramer's Rule (266), the adjoint matrix (268–269).

### 2. You need to be able to:

convert linear transformation from the row notation to the standard matrix representation and back (147)

prove (by checking the properties) or disprove (by finding a property that does not hold) that a given set is a vector space (182–184) or a subspace of a vector space (193–194);

find a coordinate vector of an element of a vector space relative to an ordered basis (207–208, see also Example 5 on p. 211);

check whether a given set of vectors is dependent or independent (by finding their coordinate vectors relative to some standard basis and then using row reduction, see Examples 3 and 4 on pp. 208–210);

prove or disprove (by checking the preservation of addition and scalar multiplication) that a transformation is linear (214–216);

write down the matrix representation of a linear transformation relative to given ordered bases (224)

compute the area of a parallelogram in  $\mathbb{R}^2$  (239–240) and  $\mathbb{R}^3$  (243–244);

find a vector perpendicular to two given vectors in  $\mathbb{R}^3$  (241–242);

compute the volume of a box in  $\mathbb{R}^3$  (244–246);

compute determinants by using row reduction (263–265) or by clever use of the properties (Example 2 on p. 265; the point is to create zeroes wherever it is easier);

solve linear systems by Cramer's Rule (266–267), invert matrices using adjoints (269–270);

find the volume change factor of a linear transformation (278) and use it for volume computations (278–280).