

# Math 22a Review Sheet for the Final Exam

Below you will find the list of what you are supposed to know in addition to topics listed in the first two review sheets. References are made to the corresponding pages in the book. The final exam problems will be modelled over those from homeworks, quizzes, and midterm exams.

## 1. You need to understand what are:

an eigenvalue and an eigenvector of a matrix (288) and a linear transformation (297), the eigenspace corresponding to an eigenvalue (296), characteristic polynomial (291), diagonalizable matrices (307); similar matrices (310), algebraic and geometric multiplicities of an eigenvalue (312), a criterion for diagonalization (313, in the modified form reflecting the fact that we were not talking about non-real eigenvalues: *a matrix  $A$  is diagonalizable (over  $\mathbb{R}$ ) if and only if all the roots of its characteristic polynomial are real, and the algebraic multiplicity of each root is equal to its geometric multiplicity as an eigenvalue of  $A$* );

the projection of a vector onto a subspace (327 for one-dimensional subspaces, 332 for the general case), the orthogonal complement of a subspace (329), orthogonal (338) and orthonormal (340) sets/bases, the Gram-Schmidt orthogonalization process (341–343),  $QR$  factorization (344), orthogonal matrices (350).

## 2. You need to be able to:

find the characteristic polynomial (291), all eigenvalues (290–293) and the corresponding eigenvectors of a matrix (293–295); find algebraic and geometric multiplicities of eigenvalues of matrices (312, Example 5), determine whether a matrix is diagonalizable (307–312); given a basis consisting of eigenvectors of  $A$  (an eigenbasis), write  $A$  in the form  $BDB^{-1}$ , where  $D$  is diagonal and  $B$  is invertible (307–309); compute  $A^k$  (by diagonalizing  $A$ , 307–309), or compute  $A^k\mathbf{x}$  for a given  $\mathbf{x} \in \mathbb{R}^n$  (by writing  $\mathbf{x}$  as a linear combination of eigenvectors, 317–318), and use it for studying recurrence relations (318–320);

compute the projection of a vector  $\mathbf{v}$  onto: the span of another vector (327–328), a subspace  $W$  (333–334 and 339–340); the latter problem might be solvable in many ways, such as:

- proof of Theorem 6.1 (333)
- if  $W^\perp$  is low-dimensional (best of all, one-dimensional) – projecting  $\mathbf{v}$  onto  $W^\perp$  and subtracting the result from  $\mathbf{v}$ ,
- if an orthogonal basis of  $W$  or  $W^\perp$  is given – using it as in Examples 2 and 3 on pp. 339–340,
- or else, it may make sense to orthogonalize the given basis and then proceed as above;

find the orthogonal complement of a given subspace  $W$  of  $\mathbb{R}^n$  (330) (note that if  $W$  is described as the null space of a matrix, then  $W^\perp$  is its row space, see 331, Illustration 3) (note also that for two-dimensional subspaces of  $\mathbb{R}^3$  the answer is given by the cross product);

find orthogonal/orthonormal basis of a subspace by applying the Gram-Schmidt orthogonalization to a given basis (341–343) (note that the latter problem might be an ingredient for computing projections, as described earlier).