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Math 28a, Quiz 1, Groups and Subgroups

1. Let G be a group, let H, K be two subgroups of G, and suppose that $G = H \cup K$. Prove that either $H = G$ or $K = G$.

Suppose not; i.e. suppose that $\exists x \in G$, $x \notin H$ (thus $x \in K$)
and also $\exists y \in G$, $y \notin K$ (thus $y \in H$)

Then consider xy; since $G = H \cup K$, either $xy \in H$ or $xy \in K$

$$\left.\begin{array}{l}\text{But } xy \in H \;\&\; y \in H \text{ implies } x \in H \\ \text{and } xy \in K \;\&\; x \in K \text{ implies } y \in K\end{array}\right\} \text{ a contradiction in either case.}$$

2. Let x and y be elements of a group G, let $z = xy$, and suppose that z belongs to the centralizer of x in G. Prove that x and y commute, that is, $yx = z$ as well.

Given: $x(xy) = (xy)x$, or $xxy = xyx$;
multiplying by x^{-1} on left, we conclude $xy = yx$.