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Math 28a, Quiz 2, Cyclic Groups

1. Let G be a cyclic group generated by a, and suppose that $|a^{14}| = 20$ and $|a^{16}| = 5$. How many elements does G have? Justify your answer.

$(a^{14})^{20} = a^{280} = e = (a^{16})^5 = a^{80} \Rightarrow a^{\gcd(280,80)} = e$

So $a^{40} = e$, which implies that $|a|$ is a factor of 40

However, $|a|$ is at least 20, since $|a^{14}| = 20$.

Moreover, $|a| = 20$ would imply $|a^{14}| = \frac{20}{\gcd(14,20)} = 10$,

a contradiction $\Rightarrow |a| = 40$, so G has 40 elements

2. Let G be a group with exactly 18 elements of order 14. How many cyclic subgroups of order 14 does G have? Justify your answer.

Each cyclic subgroup of order 14 has $\varphi(14) = 6$ elements of order 14 $\Rightarrow$ there must be $\frac{18}{6} = 3$ subgroups