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Math 28a, Quiz 3, Permutation Groups

1. Let $n \in \mathbb{N}$, let G be a subgroup of S_n of order 2010, and let $\alpha \in G$ be an odd permutation. Prove that G contains exactly 1005 odd permutations.

Consider the map $T_\alpha : G \to G$, $x \mapsto \alpha x$

It is a bijection and sends even permutations to odd ones,

and vice versa $\Rightarrow$ G has as many even permutations as odd ones

(This is analogous to the proof of Theorem 5.7.)

2. Consider the following element of S_8, written in the cycle notation:

$$\sigma = (143758)(257)(638)$$

(a) Write it as a product of disjoint cycles.

(b) What is the order of σ?

(c) Is σ odd or even? justify your answer.

In the table format: $\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 1 & 3 & 5 & 7 & 2 & 6 \end{bmatrix}$

$\Rightarrow \sigma = (143)(2867)$

$|\sigma| = \text{lcm}(3,4) = 3 \cdot 4 = 12$

σ is odd since 3-cycles are even and 4-cycles are odd