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Math 28a, Quiz 4, Isomorphisms and Cosets

1. Let G be a finite Abelian group, and let p be a prime number such that the order of G is NOT divisible by p. Consider a map ϕ from G to itself given by $\phi(x) = x^p$. Prove that ϕ is an automorphism of G.

Operation preserving: $\varphi(xy) = (xy)^p = x^p y^p = \varphi(x)\varphi(y)$
since G is Abelian

Injective: $x^p = y^p \Leftrightarrow (xy^{-1})^p = e \Leftrightarrow xy^{-1} = e \Leftrightarrow x = y$

because G has no elements of order p in view of the divisibility assumption

Surjective, since G is finite, any injective map $G \to G$ is also surjective

2. Let H, K be two subgroups of G and let $J = H \cap K$. Show that for any $a \in G$, the coset aJ is equal to the intersection of cosets aH and aK.

$x \in aJ \Leftrightarrow a^{-1}x \in J \Leftrightarrow a^{-1}x \in H$ and $a^{-1}x \in K \Leftrightarrow x \in aH$ and $x \in aK$

$\Updownarrow$

$x \in aH \cap aK$