

Your name: D. Kleinbock

Math 28a, Quiz 5, External Direct Products

Let $G_1 = S_3$ (the group of all permutations of $\{1, 2, 3\}$), $G_2 = D_4$ (the group of all symmetries of a square), and let $G = G_1 \oplus G_2$.

1. What is the order of G ? Explain.

$$|G_1| = 6, |G_2| = 8 \Rightarrow |G_1 \oplus G_2| = 48$$

2. Find (with proof) the largest natural number k such that G has an element of order k . How many elements of G of order k are there? Justify your answer.

Orders of elements of G_1 : 1, 2, 3 (2 elements of order 3)
Orders of elements of G_2 : 1, 2, 4 (2 elements of order 4)

Since $|(g_1, g_2)| = \text{lcm}(|g_1|, |g_2|)$, the maximal order is $12 = \text{lcm}(3, 4)$, and there are $2 \cdot 2 = 4$ possibilities.

3. Prove that G is not isomorphic to a subgroup of S_6 .

S_6 has no elements of order 12 \Rightarrow it is not possible to find $\varphi: G \rightarrow S_6$ which is bijective and operation-preserving