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**Math 28a, Quiz 6, Normal Subgroups and Factor Groups**

**1.** Let $G$ be a group and let $a$ be an element of $G$. Prove that $\langle a\rangle$ is a normal subgroup of $G$ if and only if for any $x \in G$ there exists $k \in \mathbb{Z}$ such that $xa = a^k x$.

$\langle a \rangle \triangleleft G \Rightarrow xax^{-1} \in \langle a \rangle \ \forall x \in G$

$\Rightarrow xax^{-1}$ is equal to $a^k$ for some $k \in \mathbb{Z}$

If $\forall x \ \exists k$ s.t. $xa = a^k x$, then $\forall x \in G \ \forall \ell \in \mathbb{Z}$,

$xa^\ell x^{-1} = a^{k\ell} \in \langle a \rangle \Rightarrow \langle a \rangle \triangleleft G$

**2.** Let $G$ be a group and let $H$ be its normal subgroup with the following property: for every $a, b \in G$, the element $aba^{-1}b^{-1}$ belongs to $H$. Prove that the factor group $G/H$ is abelian.

Need to show: $\forall a, b \in G$,

$(aH)(bH) = (ab)H = (ba)H = (bH)(aH)$

The middle equality is equivalent to

$(ab)^{-1}ba = b^{-1}a^{-1}ba \in H$, which is given.