Your name: D. Kleinbock

Math 28a, Quiz 7, Homomorphisms

1. Let $\varphi : G \to \bar{G}$ be a group homomorphism, and suppose that its image has n elements, that is, the order of $\varphi(G)$ is equal to n. Prove that $a^n \in \operatorname{Ker}\varphi$ for every $a \in G$.

$$|\varphi(G)| = n \Rightarrow (\varphi(a))^n = \bar{e} \quad \forall a \in G$$

$$\Rightarrow \varphi(a^n) = \bar{e} \quad \forall a \in G \iff a^n \in \operatorname{Ker}\varphi \quad \forall a \in G$$

2. Show that there is no homomorphism from $Z_8 \oplus Z_2 \oplus Z_2$ onto $Z_4 \oplus Z_4$.

Let φ be such a homomorphism, then $Z_4 \oplus Z_4 \cong (Z_8 \oplus Z_2 \oplus Z_2)/\operatorname{Ker}\varphi$

However the element $(4,0,0)$ must be in the kernel of φ (because all elements of $Z_4 \oplus Z_4$ have orders 4 or less, and

$\varphi(4,0,0) = \varphi(4(1,0,0)) = 4\varphi(1,0,0) = (0,0,0)$)

Therefore $(Z_8 \oplus Z_2 \oplus Z_2)/\operatorname{Ker}\varphi \cong Z_4 \oplus Z_2 \oplus Z_2$,

which is <u>not</u> isomorphic to $Z_4 \oplus Z_4$

(by counting elements of order 2 or 4)