2. $2 \cdot 3^2 \cdot 7$; $2^3 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$

4. $s = -3$, $t = 2$; $s = 8$, $t = -5$

52. $a - a = 0$; if $a - b$ is an integer k then $b - a$ is the integer $-k$; if $a - b$ is the integer n and $b - c$ is the integer m, then $a - c = (a - b) + (b - c)$ is the integer $n + m$. The set of equivalence classes is $\{[k] | \ 0 \leq k < 1, \ \ k \text{ is real}\}$. The equivalence classes can be represented by the real numbers in the interval $[0, 1)$. For any real number a, $[a] = \{a + k | \text{ where } k \text{ ranges over all integers}\}$.

54. Obviously, $a + a = 2a$ is even and $a + b$ is even implies $b + a$ is even. If $a + b$ and $b + c$ are even, then $a + c = (a + b) + (b + c) - 2b$ is also even. The equivalence classes are the set of even integers and the set of odd integers.

10. reflection.

14. Rotations of 0° and 180°; Rotations of 0° and 180° and reflections about the diagonals.

4. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

8. The identity is 25.

30. In $D_4, HR_{90}V = DR_{90}H$ but $HV \neq DH$.

8. If $x^4 = e$, then $e = (x^4)^2 = x^8 = x^6 \cdot x^2 = x^2$. If $x^5 = e$, then $e = (x^5)^2 = x^{10} = x^6 \cdot x^4 = x^4$. $|x| = 3$ or 6.

34. Note that $b^4 = abaaba = ab^2a = a(aba)a = b$. Thus, $b^3 = e$. On the other hand, $b^2 \neq e$ for if so then $b^3 = b^2$ implies that $b = e$.

50. $\{1, 9, 11, 19\}$

18. 1.

54. Mimic Exercise 53.

44. One possibility is $\langle(1234)(5678)\rangle$.

4. $U(8)$ is not cyclic while $U(10)$ is.

34. For any $x \in C(a)$ let $\phi(x) = gxg^{-1}$. To verify that $gxg^{-1} \in C(gag^{-1})$ observe that $(gxg^{-1})(gag^{-1}) = (gag^{-1})(gxg^{-1})$ if and only if $gxag^{-1} = gaxg^{-1}$ if and only if $xa = ax$. So, ϕ is a function from $C(a)$ to $C(gag^{-1})$. To see that ϕ is onto we let $y \in C(gag^{-1})$ and observe that $\phi(g^{-1}yg) = g(g^{-1}yg)g^{-1} = y$ and that $(g^{-1}yg)a = a(g^{-1}yg)$ if and only if $g(g^{-1}yg)a)g^{-1} = g(a(g^{-1}yg)g^{-1}$ if and only if $ygag^{-1} = gag^{-1}y$ if and only if $y(gag^{-1}) = (gag^{-1})y$ if and only if $y \in C(gag^{-1})$. This proves that ϕ is onto. Finally $\phi(xy) = gxyg^{-1} = gxg^{-1}gyg^{-1} = \phi(x)\phi(y)$ so ϕ is an isomorphism.

20. Since $|H \cap K|$ must divide 12 and 35, $|H \cap K| = 1$.

18. Observe that $Z_9 \oplus Z_4 \approx Z_4 \oplus Z_9 \approx \langle 3 \rangle \oplus \langle 2 \rangle$.

24. D_6. Since $S_3 \oplus Z_2$ is non-Abelian, it must be isomorphic to A_4 or D_6. But $S_3 \oplus Z_2$ contains an element of order 6 and A_4 does not.

26. $\{(0,0),(2,1)\}$.

32. $\langle (4,0,5) \rangle$

38. 4

44. $Z_{10} \oplus Z_{12} \oplus Z_6 \approx Z_2 \oplus Z_5 \oplus Z_{12} \oplus Z_6 \approx Z_2 \oplus Z_{60} \oplus Z_6 \approx Z_{60} \oplus Z_6 \oplus Z_2$.