Spring 2006, MATH 311B – TOPICS IN ANALYSIS, aka

DYNAMICS AND NUMBER THEORY ON HOMOGENEOUS SPACES

Lecturer: Dmitry Kleinbock

Class meets at: MWTh 12:10 – 1 PM, in Goldsmith 101

What it is about: The idea of the course is to give a self-contained exposition of some *recent progress* in applying dynamics on homogeneous spaces to number theory. This is a relatively modern circle of ideas which does not require a lot of background and leads to many open questions.

Prerequisites: Regardless of its 300-number, this will be an *introductory* course, accessible to beginning graduate students. Knowledge of analysis within a first-year graduate course is enough, and all participants of this year's class are invited. Some familiarity with manifolds, Lie groups, Lie algebras and Riemannian geometry will be helpful but not required, we will define and discuss all the relevant concepts.

Textbook: In the beginning we will loosely follow the following source: *Ergodic Theory and Topological Dynamics of Group Actions on Homogeneous Spaces* by Bekka and Mayer, Cambridge Univ. Press 2000. (It will be placed on reserve at the Science Library and is now available at the Math Department office as a designated textbook for Math 311b.) After that I'll discuss several concepts related to simultaneous rational approximations (badly approximable vectors, Diophantine exponents and Dirichlet constants of vectors and measures) and show how knowledge of dynamics of certain actions can help us prove theorems in number theory. This stuff is scattered around a bunch of recent papers, some not written yet. Everything will be explained in full detail.

Approximate syllabus:

- measure-preserving systems, ergodicity, mixing, ergodic theorems
- Lie groups, discrete subgroups, homogeneous spaces; $\mathrm{SL}_2(\mathbb{R})$ and hyperbolic surfaces, geodesic and horocycle flows; $\mathrm{SL}_n(\mathbb{R})$ and the space of lattices in $\mathbb{R}^n$, Mahler's Criterion, Siegel sets, unipotent and diagonal actions
- Howe-Moore's Theorem, applications to uniform distribution of horocycles, further applications to Diophantine properties of vectors in $\mathbb{R}^n$
- nondivergence of unipotent flows on the space of lattices, quantitative generalizations, applications to Diophantine exponents and Dirichlet constants of measures
- other topics – if time allows