1. Prove that if $p$ and $p^2 + 2$ are primes, then $p^3 + 2$ is also prime.

2. Find all integral solutions of $x + y = x^2 - xy + y^2$.

3. For positive integers $a, b, c, d, n$, show that if $ab = cd$, then $a^n + b^n + c^n + d^n$ is not a prime.

4. Prove that it is possible to choose $2^k$ different numbers from $0, 1, \ldots, 3^k - 1$ so that three numbers in arithmetic progression will not occur.

5. Consider the sequence $A = \{31, 331, 3331, \ldots \}$. Note that the first few elements of $A$ are prime. Prove that there are infinitely many composite numbers in $A$, and that in fact composite numbers form a subset of $A$ of positive lower density.

6. The powers $2^n$ and $5^n$ start with the same digit $d$. What are possible values of $d$?

7. Show that $y^2 = x^3 + 7$ has no integral solutions.

8. Show that no prime can be written as a sum of two squares in two different ways.

9. Can the product of (a) three (b) four (c) more than four consecutive integers be equal to a (more than first) power of an integer?

10. Show that if $4^n + 2^n + 1$ is a prime, then $n$ is a power of 3.

11. Prove that there are infinitely many powers of 2 in the sequence $\lfloor n\sqrt{2} \rfloor$.

12. Do there exist positive integers $x, y$ such that $x + y$, $2x + y$ and $x + 2y$ are perfect squares?