Math 326a, Fall 2006, Problem Set # 8

Games!!!

1. A and B alternately draw diagonals of a regular 2006-gon. They may connect two vertices if the diagonal does not intersect an earlier drawn one. The loser is the one who cannot move. Who wins?

2. A and B start with $p = 1$. Then they alternately multiply p by one of the numbers 2 to 9. The winner is the one who first reaches (a) $p \geq 1000$ (b) $p \geq 10^6$. Who wins, A or B?

3. A crosses out any 128 of the numbers $0, 1, \ldots, 256$. Then B crosses out any 64 of the remaining numbers, then A crosses out any 32 of the numbers, and so on until finally B crosses out 1 number. Since $128 + \cdots + 1 = 255$, there will be two numbers a and b left. B pays the difference $|b - a|$ to A. How should A play in order to get as much as possible? How should B play to lose as little as possible? How much does A win per game if both players use their optimal strategies?

4. In the equation $x^3 + \_\_x^2 + \_\_x + \_\_ = 0$, A replaces one of the three __s by a nonzero integer. Then B replaces one of the two remaining __s by a nonzero integer, and finally A replaces the last __ by a nonzero integer. Prove that A can play so that all three roots of the resulting cubic equation are integers.

5. A and B alternately replace the stars in the polynomial $x^{10} + *x^9 + \cdots + *x + 1$ by real numbers. If the resulting polynomial has no real roots, then A wins, otherwise B wins. Can B win regardless of how A plays?

6. A and B alternately write positive integers $\leq p$ on the blackboard. Writing divisors of numbers that are already written is not allowed. The one who cannot make a move loses. Who wins for (a) p = 10 (b) p = 1000?

7. A places a knight onto an 8×8 board. Then B makes a legal chess move. Then A makes a legal chess move with the restriction that a square visited before cannot be visited again, and so on. The one who cannot make a move loses. Who wins?

8. Let n be a positive integer and $M = \{1, 2, 3, 4, 5, 6\}$. A starts with any digit from M. Then B appends to it a digit from M, and so on, until they get a number with $2n$ digits. If the result is a multiple of 9, then B wins, otherwise A wins. Who wins, depending on n?

9. Start with two piles of p and q chips, respectively. A and B move alternately. A move consists in removing one of the piles and splitting the other pile into two piles. The one who cannot make a move loses. Who wins, depending on the initial conditions?

10. A and B alternately move a knight on an 8×8 (or $2n \times 2n$) board. A makes only horizontal moves $(x, y) \mapsto (x \pm 2, y \pm 1)$, and B makes only vertical moves $(x, y) \mapsto (x \pm 1, y \pm 2)$. A starts by choosing a square and making a move. Visiting a square for a second time is not permitted. The loser is the one who cannot move. Find a winning strategy for A.