Math 36a Fall 2008 HW1 Answers & Solutions

(Chapter 1) Points.

P 1. (a) By the generalized basic principle of counting there are

$$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 67{,}600{,}000$$

(b) $26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 19{,}656{,}000$

6 (3 each)

4. There are 4! possible arrangements. By assigning instruments to Jay, Jack, John and Jim, in that order, we see by the generalized basic principle that there are $2 \cdot 1 \cdot 2 \cdot 1 = 4$ possibilities. 4

8. (a) $5! = 120$

(b) $\frac{7!}{2!2!} = 1260$

(c) $\frac{11!}{4!4!2!} = 34{,}650$

(d) $\frac{7!}{2!2!} = 1260$

8 (2 each)

11. (a) $6!$

(b) $3!2!3!$

(c) $3!4!$

6 (2 each)

15. There are $\binom{10}{5}\binom{12}{5}$ possible choices of the 5 men and 5 women. They can then be paired up in 5! ways, since if we arbitrarily order the men then the first man can be paired with any of the 5 women, the next with any of the remaining 4, and so on. Hence, there are $5!\binom{10}{5}\binom{12}{5}$ possible results. 4

19. (a) There are $\binom{8}{3}\binom{4}{3}+\binom{8}{3}\binom{2}{1}\binom{4}{2} = 896$ possible committees. 3

There are $\binom{8}{3}\binom{4}{3}$ that do not contain either of the 2 men, and there are $\binom{8}{3}\binom{2}{1}\binom{4}{2}$ that contain exactly 1 of them.

(b) There are $\binom{6}{3}\binom{6}{3}+\binom{2}{1}\binom{6}{2}\binom{6}{3} = 1000$ possible committees. 3

(c) There are $\binom{7}{3}\binom{5}{3}+\binom{7}{2}\binom{5}{3}+\binom{7}{3}\binom{5}{2} = 910$ possible committees. There are $\binom{7}{3}\binom{5}{3}$ in which neither feuding party serves; $\binom{7}{2}\binom{5}{3}$ in which the feuding women serves; and $\binom{7}{3}\binom{5}{2}$ in which the feuding man serves. 4

22. There are $\frac{4!}{2!2!}$ paths from A to the circled point; and $\frac{3!}{2!1!}$ paths from the circled point to B. Thus, by the basic principle, there are 18 different paths from A to B that go through the circled point. 6

30. $2 \cdot 9! - 2^2 8!$ since $2 \cdot 9!$ is the number in which the French and English are next to each other and $2^2 8!$ the number in which the French and English are next to each other and the U.S. and Russian are next to each other. 6

T 7. $$\binom{n-1}{r}+\binom{n-1}{r-1} = \frac{(n-1)!}{r!(n-1-r)!}+\frac{(n-1)!}{(n-r)!(r-1)!}$$

$$= \frac{n!}{r!(n-r)!}\left[\frac{n-r}{n}+\frac{r}{n}\right] = \binom{n}{r}$$

10

60