15. (a) $\Phi(.8333) = .7977$
(b) $2\Phi(1) - 1 = .6827$
(c) $1 - \Phi(.3333) = .3695$
(d) $\Phi(1.6667) = .9522$
(e) $1 - \Phi(1) = .1587$

5 (1 each)

18. $.2 = P\left\{\frac{X-5}{\sigma} > \frac{9-5}{\sigma}\right\} = P\{Z > 4/\sigma\}$ where Z is a standard normal. But from the normal table $P\{Z < .84\} \approx .80$ and so

$.84 \approx 4/\sigma$ or $\sigma \approx 4.76$

That is, the variance is approximately $(4.76)^2 = 22.66$.

5

20. Let X denote the number in favor. Then X is binomial with mean 65 and standard deviation $\sqrt{65(.35)} \approx 4.77$. Also let Z be a standard normal random variable.

(a) $P\{X \ge 50\} = P\{X \ge 49.5\} = P\{X - 65\}/4.77 \ge -15.5/4.77$
$\approx P\{Z \ge -3.25\} \approx .9994$

(b) $P\{59.5 \le X \le 70.5\} \approx P\{-5.5/4.77 \le Z \le 5.5/4.77\}$
$= 2P\{Z \le 1.15\} - 1 \approx .75$

(c) $P\{X \le 74.5\} \approx P\{Z \le 9.5/4.77\} \approx .977$

6 (2 each)

23. (a) $$P\{149.5 < X < 200.5\} = P\left\{\frac{149.5 - \frac{1000}{6}}{\sqrt{1000\frac{1}{6}\frac{5}{6}}} < Z < \frac{200.5 - \frac{1000}{6}}{\sqrt{1000\frac{1}{6}\frac{5}{6}}}\right\}$$
$$= \Phi\left(\frac{200.5 - 166.7}{\sqrt{5000/36}}\right) - \Phi\left(\frac{149.5 - 166.7}{\sqrt{5000/36}}\right)$$
$$\approx \Phi(2.87) + \Phi(1.46) - 1 = .9258.$$

4

(b) $$P\{X < 149.5\} = P\left\{Z < \frac{149.5 - 800(1/5)}{\sqrt{800\frac{1}{5}\frac{4}{5}}}\right\}$$
$$= P\{Z < -.93\}$$
$$= 1 - \Phi(.93) = .1762.$$

4

25. Let X denote the number of unacceptable items among the next 150 produced. Since X is a binomial random variable with mean $150(.05) = 7.5$ and variance $150(.05)(.95) = 7.125$, we obtain that, for a standard normal random variable Z.

$$P\{X \le 10\} = P\{X \le 10.5\}$$
$$= P\left\{\frac{X - 7.5}{\sqrt{7.125}} \le \frac{10.5 - 7.5}{\sqrt{7.125}}\right\}$$
$$\approx P\{Z \le 1.1239\}$$
$$= .8695$$

5

2. (a) $p(0,0) = \dfrac{8\cdot 7}{13\cdot 12} = 14/39,$

$p(0,1) = p(1,0) = \dfrac{8\cdot 5}{13\cdot 12} = 10/39$

$p(1,1) = \dfrac{5\cdot 4}{13\cdot 12} = 5/39$ 5

3. (a) $p(0,0) = (10/13)(9/12) = 15/26$

$p(0,1) = p(1,0) = (10/13)(3/12) = 5/26$ 5

$p(1,1) = (3/13)(2/12) = 1/26$

8. $f_Y(y) = c\displaystyle\int_{-y}^{y}(y^2 - x^2)e^{-y}dx$

$= \dfrac{4}{3}cy^3e^{-y}, -0 < y < \infty$

$\displaystyle\int_0^\infty f_Y(y)dy = 1 \Rightarrow c = 1/8$ and so $f_Y(y) = \dfrac{y^3e^{-y}}{6}, 0 < y < \infty$

$f_X(x) = \dfrac{1}{8}\displaystyle\int_{|x|}^{\infty}(y^2 - x^2)e^{-y}dy$

$= \dfrac{1}{4}e^{-|x|}(1+|x|)$ upon using $-\int y^2e^{-y} = y^2e^{-y} + 2ye^{-y} + 2e^{-y}$ 9 (3 for each part)

9. (b) $f_X(x) = \dfrac{6}{7}\displaystyle\int_0^2\left(x^2 + \dfrac{xy}{2}\right)dy = \dfrac{6}{7}(2x^2 + x)$

(c) $P\{X > Y\} = \dfrac{6}{7}\displaystyle\int_0^1\int_0^x\left(x^2 + \dfrac{xy}{2}dydx\right) = \dfrac{15}{56}$

(d) $P\{Y > 1/2 \mid X < 1/2\} = P\{Y > 1/2, X < 1/2\}/P\{X < 1/2\}$

$$= \frac{\displaystyle\int_{1/2}^{2}\int_0^{1/2}\left(x^2 + \frac{xy}{2}dxdy\right)}{\displaystyle\int_0^{1/2}(2x^2 + x)dx}$$ 10

60