(b) $p(0, 0, 0) = (10/13)(9/12)(8/11) = 60/143$

$p(0, 0, 1) = p(0, 1, 0) = p(1, 0, 0) = (10/13)(9/12)(3/11) = 45/286$

$p(i, j, k) = (3/13)(2/12)(10/11) = 5/143$ if $i + j + k = 2$

$p(1, 1, 1) = (3/13)(2/12)(1/11) = 1/286$ ← 2

(b) $p(0, 0, 0) = \dfrac{8\cdot7\cdot6}{13\cdot12\cdot11} = 28/143$

$p(0, 0, 1) = p(0, 1, 0) = p(1, 0, 0) = \dfrac{8\cdot7\cdot5}{13\cdot12\cdot11} = 70/429$

$p(0, 1, 1) = p(1, 0, 1) = p(1, 1, 0) = \dfrac{8\cdot5\cdot4}{13\cdot12\cdot11} = 40/429$

$p(1, 1, 1) = \dfrac{5\cdot4\cdot3}{13\cdot12\cdot11} = 5/143$ ← 2

18. $P\{Y - X > L/3\} = \displaystyle\iint_{y-x>L/3} \frac{4}{L^2}\,dydx$

$\dfrac{L}{2} < y < L$

$0 < x < \dfrac{L}{2}$

$= \dfrac{4}{L^2}\left[\displaystyle\int_0^{L/6}\int_{L/2}^{L} dydx + \int_{L/6}^{L/2}\int_{x+L/3}^{L} dydx\right]$

$= \dfrac{4}{L^2}\left[\dfrac{L^2}{12} + \dfrac{5L^2}{24} - \dfrac{7L^2}{72}\right] = 7/9$ ← 5

19. $\displaystyle\int_0^1\int_0^x \frac{1}{x}\,dydx = \int_0^1 dx = 1$ 1

(a) $\displaystyle\int_y^1 \frac{1}{x}dx = -\ln(y),\ 0 < y < 1$ 1

(b) $\displaystyle\int_0^x \frac{1}{x}dy = 1,\quad 0 < y < 1$ 1

(c) $\dfrac{1}{2}$ 1

(d) Integrating by parts gives that

$$\int_0^1 y\ \ln(y)dy = -1 - \int_0^1 (y\ln(y) - y)dy$$

yielding the result

$$E[Y] = -\int_0^1 y\ln(y)dy = 1/4$$ 2

20. (a) yes: $f_X(x) = xe^{-x}, f_Y(y) = e^{-y},\ 0 < x < \infty,\ 0 < y < \infty$ 2

(b) no: $f_X(x) = \displaystyle\int_x^1 f(x,y)dy = 2(1-x),\ 0 < x < 1$

$f_Y(y) = \displaystyle\int_0^y f(x,y)dx = 2y,\ 0 < y < 1$ 2

21. (a) We must show that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$. Now,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = \int_0^1\int_0^{1-y} 24xy\ dxdy$$
$$= \int_0^1 12y(1-y)^2dy$$
$$= \int_0^1 12(y - 2y^2 + y^3)dy$$
$$= 12(1/2 - 2/3 + 1/4) = 1$$ 2

(b) $$E[X] = \int_0^1 xf_X(x)dx$$
$$= \int_0^1 x\int_0^{1-x} 24xy\ dydx$$
$$= \int_0^1 12x^2(1-x)^2dx = 2/5$$ 2

(c) 2/5 2

22. (a) No, since the joint density does not factor. 2

(b) $f_X(x) = \int_0^1 (x+y)dy = x + 1/2,\ 0 < x < 1.$ 2

(c) $P\{X+Y<1\} = \int_0^1 \int_0^{1-x} (x+y)dydx$

$= \int_0^1 [x(1-x) + (1-x)^2/2]dx = 1/3$ 3

23. (a) yes

$f_X(x) = 12x(1-x)\int_0^1 ydy = 6x(1-x),\ 0 < x < 1$

$f_Y(y) = 12y\int_0^1 x(1-x)dx = 2y,\ 0 < y < 1$

2

(b) $E[X] = \int_0^1 6x^2(1-x)dx = 1/2$ 2

(c) $E[Y] = \int_0^1 2y^2 dy = 2/3$ 2

(d) $\mathrm{Var}(X) = \int_0^1 6x^3(1-x)dx - 1/4 = 1/20$ 2

(e) $\mathrm{Var}(Y) = \int_0^1 2y^3 dy - 4/9 = 1/18$ 2

26. (a) $F_{A,B,C}(a,b,c) = abc \quad 0 < a, b, c < 1$ 3

(b) The roots will be real if $B^2 \geq 4AC$. Now

$$P\{AC \leq x\} = \iint\limits_{\substack{c \leq x/a \\ 0 \leq a \leq 1 \\ 0 \leq c \leq 1}} dadc = \int_0^x \int_0^1 dcda + \int_x^1 \int_0^{x/a} dcda$$

$$= x - x\log x.$$

Hence, $F_{AC}(x) = x - x\log x$ and so
$f_{AC}(x) = -\log x,\ 0 < x < 1$

$$P\{B^2/4 \geq AC\} = -\int_0^1 \int_0^{b^2/4} \log x dxdb$$

$$= \int_0^1 \left[\frac{b^2}{4} - \frac{b^2}{4}\log(b^2/4)\right]db$$

$$= \frac{\log 2}{6} + \frac{5}{36}$$

where the above uses the identity

$$\int x^2 \log x dx = \frac{x^3 \log x}{3} - \frac{x^3}{9}.$$ 7

27. (a) $P\{X+Y \leq a\} = \int_0^a \int_0^{a-x} e^{-y}dydx = a - 1 + e^{-a},\ a < 1$

$= \int_0^1 \int_0^{a-x} e^{-y}dydx = 1 - e^{-a}(e-1),\ a > 1$

4

(b) $P\{Y > X/a\} = \int_0^1 \int_{x/a}^{\infty} e^{-y}dydx = a(1 - e^{-1/a})$ 4

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