Chapter 6

P30. (a) e^{-2}

(b) $1 - e^{-2} - 2e^{-2} = 1 - 3e^{-2}$
The number of typographical errors on each page should approximately be Poisson distributed and the sum of independent Poisson random variables is also a Poisson random variable. 2 2

33. Let X denote Jill's score and let Y be Jack's score. Also, let Z denote a standard normal random variable.

(a)
$$\begin{aligned} P\{Y > X\} &= P\{Y - X > 0\} \\ &\approx P\{Y - X > .5\} \\ &= P\left\{\frac{Y - X - (160-170)}{\sqrt{(20)^2 + (15)^2}} > \frac{.5 - (160-170)}{\sqrt{(20)^2 + (15)^2}}\right\} \\ &\approx P\{Z > .42\} \approx .3372 \end{aligned}$$
3

(b)
$$\begin{aligned} P\{X + Y > 350\} &= P\{X + Y > 350.5\} \\ &= P\left\{\frac{X + Y - 330}{\sqrt{(20)^2 + (15)^2}} > \frac{20.5}{\sqrt{(20)^2 + (15)^2}}\right\} \\ &\approx P\{Z > .82\} \approx .2061 \end{aligned}$$
3

34. Let X and Y denote, respectively, the number of males and females in the sample that never eat breakfast. Since

$$E[X] = 50.4,\ \mathrm{Var}(X) = 37.6992,\ E[Y] = 47.2,\ \mathrm{Var}(Y) = 36.0608$$

it follows from the normal approximation to the binomial that is approximately distributed as a normal random variable with mean 50.4 and variance 37.6992, and that Y is approximately distributed as a normal random variable with mean 47.2 and variance 36.0608. Let Z be a standard normal random variable.

(a)
$$\begin{aligned} P\{X + Y \geq 110\} &= P\{X + Y \geq 109.5\} \\ &= P\left\{\frac{X + Y - 97.6}{\sqrt{73.76}} \geq \frac{109.5 - 97.6}{\sqrt{73.76}}\right\} \\ &\approx P\{Z > 1.3856\} \approx .0829 \end{aligned}$$
3

(b)
$$\begin{aligned} P\{Y \geq X\} &= P\{Y - X \geq -.5\} \\ &= P\left\{\frac{Y - X - (-3.2)}{\sqrt{73.76}} \geq \frac{-.5 - (-3.2)}{\sqrt{73.76}}\right\} \\ &\approx P\{Z \geq .3144\} \approx .3766 \end{aligned}$$
5

Chapter 7

P 1. Let $X = 1$ if the coin toss lands heads, and let it equal 0 otherwise. Also, let Y denote the value that shows up on the die. Then, with $p(i, j) = P\{X = i, Y = j\}$

$$\begin{aligned} E[\text{return}] &= \sum_{j=1}^{6} 2jp(1, j) + \sum_{j=1}^{6} \frac{j}{2} p(0, j) \\ &= \frac{1}{12}(42 + 10.5) = 52.5/12 \end{aligned}$$
6

3. $E[|X-Y|^a] = \int_0^1\int_0^1 |x-y|^a dydx$. Now

$$\int_0^1 |x-y|^a dy = \int_0^x (x-y)^a dy + \int_x^1 (y-x)^a dy$$
$$= \int_0^x u^a du + \int_0^{1-x} u^a du$$
$$= [x^{a+1} + (1-x)^{a+1}]/(a+1)$$

Hence,

$$E[|X-Y|^a] = \frac{1}{a+1}\int_0^1 [x^{a+1} + (1-x)^{a+1}]dx$$
$$= \frac{2}{(a+1)(a+2)}$$

6

5. The joint density of the point (X, Y) at which the accident occurs is

$$f(x, y) = \frac{1}{9}, -3/2 < x, y < 3/2$$
$$= f(x) f(y)$$

where

$$f(a) = 1/3, -3/2 < a < 3/2.$$

Hence we may conclude that X and Y are independent and uniformly distributed on $(-3/2, 3/2)$ Therefore,

$$E[\,|X| + |Y|\,] = 2\int_{-3/2}^{3/2} \frac{1}{3}|x|dx = \frac{4}{3}\int_0^{3/2} xdx = 3/2.$$

6

6. $E\left[\sum_{i=1}^{10} X_i\right] = \sum_{i=1}^{10} E[X_i] = 10(7/2) = 35.$

4

30. $E[(X-Y)]^2 = \text{Var}(X-Y) = \text{Var}(X) + \text{Var}(-Y) = 2\sigma^2$

4

31. $\text{Var}\left(\sum_{i=1}^{10} X_i\right) = 10\ \text{Var}(X_1)$. Now

$$\text{Var}(X_1) = E[X_1^2] - (7/2)^2$$
$$= [1 + 4 + 9 + 16 + 25 + 36]/6 - 49/4$$
$$= 35/12$$

and so $\text{Var}\left(\sum_{i=1}^{10} X_i\right) = 350/12.$

5

33. (a) $E[X^2 + 4X + 4] = E[X^2] + 4E[X] + 4 = \text{Var}(X) + E^2[X] + 4E[X] + 4 = 14$

3

(b) $\text{Var}(4 + 3X) = \text{Var}(3X) = 9\text{Var}(X) = 45$

2

36. Let $X_i = \begin{cases} 1 & \text{roll } i \text{ lands on 1} \\ 0 & \text{otherwise} \end{cases}$, $Y_i = \begin{cases} 1 & \text{roll } i \text{ lands on 2} \\ 0 & \text{otherwise} \end{cases}$

$$\text{Cov}(X_i, Y_j) = E[X_i Y_j] - E[X_i]E[Y_j]$$
$$= \begin{cases} -\frac{1}{36} & i = j \text{ (since } X_iY_j = 0 \text{ when } i = j \\ \frac{1}{36} - \frac{1}{36} = 0 & i \neq j \end{cases}$$

$$\text{Cov} \sum_i X_i, \sum_j Y_j = \sum_i \sum_j \text{Cov}(X_i, Y_j) \quad = -\frac{n}{36}$$

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