Math 36a Fall 2008 Homework 12 Solutions (Chapter 8)

P1. $P\{0 \le X \le 40\} = 1 - P\{|X-20| > 20\} \ge 1 - 20/400 = 19/20$ (4)

2. (a) $P\{X \ge 85\} \le E[X]/85 = 15/17$ (2)

(b) $P\{65 \le X \le 85) = 1 - P\{|X-75| > 10\} \ge 1 - 25/100$ (2)

(c) $P\left\{\left|\sum_{i=1}^{n} X_i/n - 75\right| > 5\right\} \le \frac{25}{25n}$ so need $n = 10$ (3)

4. (a) $P\left\{\sum_{i=1}^{20} X_i > 15\right\} \le 20/15$ (2)

(b) $P\left\{\sum_{i=1}^{20} X_i > 15\right\} = P\left\{\sum_{i=1}^{20} X_i > 15.5\right\}$

$\approx P\left\{Z > \frac{15.5-20}{\sqrt{20}}\right\}$

$= P\{Z > -1.006\}$

$\approx .8428$ (4)

5. Letting X_i denote the i^{th} roundoff error it follows that $E\left[\sum_{i=1}^{50} X_i\right] = 0$,

$\mathrm{Var}\left(\sum_{i=1}^{50} X_i\right) = 50\ \mathrm{Var}(X_1) = 50/12$, where the last equality uses that $.5 + X$ is uniform (0, 1) and so $\mathrm{Var}(X) = \mathrm{Var}(.5 + X) = 1/12$. Hence,

$P\left\{\left|\sum X_i\right| > 3\right\} \approx P\{|N(0,1)| > 3(12/50)^{1/2}\}$ by the central limit theorem

$= 2P\{N(0,1) > 1.47 = .1416$ (6)

6. If X_i is the outcome of the i^{th} roll then $E[X_i] = 7/2$ $\mathrm{Var}(X_i) = 35/12$ and so

$P\left\{\sum_{i=1}^{79} X_i \le 300\right\} = P\left\{\sum_{i=1}^{79} X_i \le 300.5\right\}$

$\approx P\left\{N(0,1) \le \frac{300.5 - 79(7/2)}{(79 \times 35/12)^{1/2}}\right\} = P\{N(0,1) \le 1.58\} = .9429$ (6)

7. $P\left\{\sum_{i=1}^{100} X_i > 525\right\} \approx P\left\{N(0,1) > \frac{525-500}{\sqrt{(100 \times 25)}}\right\} = P\{N(0,1) > .5\} = .3085$

where the above uses that an exponential with mean 5 has variance 25. (6)

13. (a) $P\{\bar{X} > 80\} = P\left\{\frac{\bar{X}-74}{14/5} > 15/7\right\} \approx P\{Z > 2.14\} \approx .0162$ (3)

(b) $P\{\bar{Y} > 80\} = P\left\{\frac{\bar{Y}-74}{14/8} > 24/7\right\} \approx P\{Z > 3.43\} \approx .0003$ (3)

(c) Using that $SD(\bar{Y} - \bar{X}) = \sqrt{196/64 + 196/25} \approx 3.30$ we have

$P\{\bar{Y} - \bar{X} > 2.2\} = P\{\bar{Y} - \bar{X}\}/3.30 > 2.2/3.30\}$

$\approx P\{Z > .67\} \approx .2514$ (4)

(d) same as in (c)

2. $P\{D > \alpha\} = P\{|X - \mu| > \alpha\mu\} \le \frac{\varsigma^2}{\alpha^2\mu^2} = \frac{1}{\alpha^2 r^2}$ (6)

14. Suppose n components are in stock. The probability they will last for at least 2000 hours is

$p = P\left\{\sum_{i=1}^{n} X_i \ge 2000\right\} \approx P\left\{Z \ge \frac{2000-100n}{30\sqrt{n}}\right\}$

where Z is a standard normal random variable. Since $.95 = P\{Z \ge -1.64\}$ it follows that $p \ge .95$ if

$\frac{2000-100n}{30\sqrt{n}} \le -1.64$

or, equivalently,

$(2000 - 100n)/\sqrt{n} \le -49.2$

and this will be the case if $n \ge 23$. (9)

/60