P1. 
\[ P(0 \leq X \leq 40) = 1 - P\left( |X - 20| > 20 \right) \geq 1 - 20/400 = 19/20 \]

2. (a) \( P(X \geq 85) \leq E[X]/85 = 15/17 \)
(b) \( P(65 \leq X \leq 85) = 1 - P\left( |X - 75| > 10 \right) \geq 1 - 25/100 \)
(c) \( P\left( \left| \frac{1}{n} \sum_{i=1}^{n} X_i - 75 \right| > 5 \right) \leq \frac{25}{25n} \) so need \( n = 10 \)

5. Letting \( X_i \) denote the \( i \)th roundoff error it follows that \( E\left[ \sum_{i=1}^{n} X_i \right] = 0, \)
\[ \text{Var}\left( \sum_{i=1}^{n} X_i \right) = 50 \text{Var}(X_i) = 50/12, \] where the last equality uses that \(.5 + X\) is uniform \((0, 1)\)
and so \( \text{Var}(X) = \text{Var}(.5 + X) = 1/12. \) Hence,
\[ P\left( \left| \sum X_i \right| > 3 \right) \approx P\left( |N(0, 1)| > 3(12/50)^{1/2} \right) \] by the central limit theorem
\[ = 2P(N(0, 1) > 1.47) = .1416 \]

6. If \( X_i \) is the outcome of the \( i \)th roll then \( E[X_i] = 7/2 \) \( \text{Var}(X) = 35/12 \) and so
\[ P\left( \sum_{i=1}^{n} X_i \leq 300 \right) = P\left( \sum_{i=1}^{n} X_i \leq 300.5 \right) \]
\[ \approx P\left( N(0, 1) \leq \frac{300.5 - 79(7/2)}{79 \times 35/12^{1/2}} \right) = P(N(0, 1) \leq 1.58) = .9429 \]

7. \( P\left( \sum_{i=1}^{100} X_i > 525 \right) \approx P\left( N(0, 1) > \frac{525 - 500}{\sqrt{100 \times 25}} \right) = .3085 \)
where the above uses that an exponential with mean 5 has variance 25.

13. (a) \( P(\overline{X} > 80) = P\left( \frac{\overline{X} - 74}{14/5} > 15/7 \right) \approx P\left( Z > 2.14 \right) \approx .0162 \)
(b) \( P(\overline{Y} > 80) = P\left( \frac{\overline{Y} - 74}{14/8} > 24/7 \right) \approx P\left( Z > 3.43 \right) \approx .0003 \)
(c) Using that \( \text{SD}(\overline{Y} - \overline{X}) = \sqrt{196/64 + 196/25} \approx 3.30 \) we have
\[ P(\overline{Y} - \overline{X} > 2.2) = P(\overline{Y} - \overline{X} / 3.30 > 2.2/3.30) \approx P\left( Z > .67 \right) \approx .2514 \)
(d) same as in (c)

14. Suppose \( n \) components are in stock. The probability they will last for at least 2000 hours is
\[ p = P\left( \sum_{i=1}^{n} X_i \geq 2000 \right) \approx P\left( Z \geq \frac{2000 - 100n}{30\sqrt{n}} \right) \]
where \( Z \) is a standard normal random variable. Since \.95 = P(Z \geq -1.64) \) it follows that \( p \geq .95 \) if
\[ \frac{2000 - 100n}{30\sqrt{n}} \leq -1.64 \]
or, equivalently,
\[ (2000 - 100n)/\sqrt{n} \leq -49.2 \]
and this will be the case if \( n \geq 23. \)