1. (20 pts) Suppose that an engine of an airplane in flight will fail with probability $1 - p$ independent of other engines of the plane. Also suppose that a plane can complete the journey successfully if at least half of its engines remain operating.

Is it true that a four-engine plane is always preferable to a two-engine plane? If yes, explain why; if no, determine for what values of $p$ a two-engine plane is preferable.

For the two-engine plane, the probability of success is

$$P_2 = 1 - (1-p)^2,$$

for the four-engine plane it is

$$P_4 = 1 - (1-p)^4 - 4p(1-p)^3;$$ their difference

$$P_2 - P_4 = (1-p)^4 + 4p(1-p)^3 - (1-p)^2 = (1-p)^2 \left[ (1-p)^2 + 4p(1-p) - 1 \right]\]

$$= (1-p)^2 \left[ 1 - 2p + p^2 + 4p - 4p^2 - 1 \right] = p(1-p)^2 (2-3p) < 0$$

If and only if $\frac{2}{3} < p < 1$; this is the range of $p$'s for which four-engine plane is preferable, while two-engine one is preferable for $0 < p < \frac{2}{3}$.

If $p = 0, \frac{2}{3}$ or 1, there is no difference.