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Math 36a, Fall 2008, Quiz # 1

1. A delicatessen has advertised that it offers over 1000 varieties of sandwiches. If at this deli it is possible to have any combination of salami, turkey, bologna, corned beef, ham and cheese on French bread with possible additions of lettuce, tomato, onions and mayonnaise, is the deli's advertisement true? Assume that a sandwich necessarily has bread and at least one type of meat or cheese.

For each sandwich, each of the 10 ingredients may or may not be present, giving $2^{10} = 1024$ possibilities. However, $2^4 = 16$ of them (with no meat/cheese) are not allowed \Rightarrow total is $1024 - 16 = 1008 > 1000$

A more complicated way: add all the binomial coefficients,

$$\left[\binom{6}{1} + \binom{6}{2} + \dots + \binom{6}{6} \right] \left[\binom{4}{0} + \dots + \binom{4}{4} \right] = 63 \cdot 16 = 1008$$

2. Suppose that 6 women and 2 men enter a restaurant and sit at random around a table that has 4 chairs on one side and another 4 on the other side. What is the probability that the men are not sitting on one side?

The shortest solution: for any seat choice of the first man, there are 3 seats left on his side and 4 on the opposite side, each equally likely to be occupied by the second man \Rightarrow the probability is $\frac{4}{7}$

Other solutions:

$$\frac{\text{\# of ways for the pair of men to sit on the opposite sides}}{\text{\# of all ways for them to sit}} = \frac{4 \cdot 4}{\binom{8}{2}} = \frac{16}{28} = \frac{4}{7}, \text{ or}$$

$$\frac{\text{\# of arrangements with men sitting at the same side}}{\text{\# of all arrangements}} = \frac{2 \cdot \binom{6}{4} \cdot 4! \cdot 4!}{8!} = \frac{3}{7}$$