1. In a game, Emily gives Harry three well-balanced quarters to flip. Harry will get to keep all the ones that will land heads. He will return those landing tails. However, if all three coins land tails, Harry must pay Emily two dollars. Find the expected value and the variance of Harry's net gain.

\[
\begin{array}{c|c|c|c|c}
   & -2 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\
\hline
p(x) & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8}
\end{array}
\]

\[
E[X] = \sum xp(x) = (-2)\frac{1}{8} + \frac{1}{4} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{3}{8} + \frac{3}{4} \cdot \frac{1}{8} = \frac{1}{8}
\]

\[
E[X^2] = \sum x^2 p(x) = (-2)^2 \frac{1}{8} + (\frac{1}{4})^2 \frac{3}{8} + (\frac{1}{2})^2 \frac{3}{8} + (\frac{3}{4})^2 \frac{1}{8} = \frac{1}{2} + \frac{3}{128} + \frac{3}{32} + \frac{9}{128} = \frac{89}{128} = \frac{11}{16}
\]

\[
\text{Var}[X] = E[X^2] - E[X]^2 = \frac{89}{128} - \frac{1}{8} = \frac{43}{64}
\]

2. Suppose that 2.5% of the population of a border town are illegal immigrants. Find the probability that in a theater of this town with 80 random viewers, there are at least two illegal immigrants. Use Poisson approximation, and express your answer in terms of powers of e.

The number $X$ of illegal immigrants in the theater is binomial with $p = 0.025$ and $n = 80 \Rightarrow$ approximated by Poisson with $\lambda = np = 0.025 \cdot 80 = 2$

\[
P(X \geq 2) = 1 - P(X = 0) - P(X = 1) \approx 1 - e^{-2} \cdot \frac{2^0}{0!} - e^{-2} \cdot \frac{2^1}{1!} = 1 - 3e^{-2}
\]