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Math 36a, Fall 2008, Quiz # 4

1. The time it takes for a student to finish an aptitude test (in hours) has a density function of the form

$$f(x) = \begin{cases} c(x-1)(2-x) & \text{if } 1 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Determine the constant c.

(b) What is the probability that a student will finish the aptitude test in less than 75 minutes?

(a) $\int_1^2 (x-1)(2-x)\,dx = \int_1^2 (-x^2+3x-2)\,dx = \left[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right]_1^2 =$

$= -\frac{8}{3} + 6 - 4 + \frac{1}{3} - \frac{3}{2} + 2 = \frac{1}{6} \Rightarrow c = 6$

(b) $P(x < 1\frac{1}{4}) = \int_1^{5/4} f(x)\,dx = \left[-2x^3 + 9x^2 - 12x\right]_1^{5/4} =$

$= -\frac{125}{32} + \frac{225}{16} - 15 + 2 - 9 + 12 = \frac{325}{32} - 10 = \frac{5}{32} = .15625$

2. Let X be an exponential random variable with parameter λ. Calculate $E[e^X]$. For which values of λ is $E[e^X]$ finite?

$$E[e^x] = \int_0^\infty e^x \lambda e^{-\lambda x}\,dx = \lambda \int_0^\infty e^{(1-\lambda)x}\,dx$$

$$= \frac{\lambda}{\lambda - 1} \quad \text{if } 1-\lambda < 0,\ \lambda > 1$$