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Math 36a, Fall 2008, Quiz # 6

1. Let X, Y, and Z be three independent random variables such that $E(X) = E(Y) = E(Z) = -1$ and $\mathrm{Var}(X) = \mathrm{Var}(Y) = \mathrm{Var}(Z) = 1$. Calculate $E\left[X^2(Y+5Z)^2\right]$.

$$E[X^2(Y+5Z)^2] = E[X^2(Y^2+10YZ+25Z^2)] = E[X^2Y^2+10X^2YZ+25X^2Z^2]$$

$$= E[X^2Y^2]+E[10X^2YZ]+E[25X^2Z^2] \underset{(\text{indep})}{=} E[X^2]E[Y^2]+10E[X^2]E[Y]E[Z]+$$

$$+25E[X^2]E[Z^2] = 2\cdot 2+10\cdot 2(-1)(-1)+25\cdot 2\cdot 2 = 124$$

since $E[X^2] = \mathrm{Var}[X] + E[X]^2 = 1+1=2$, same for $E[Y^2]$ and $E[Z^2]$.

2. In n independent Bernoulli trials, each with probability of success p, let X be the number of successes and Y be the number of failures. Calculate $E(XY)$ and $\mathrm{Cov}(X,Y)$.

Since $X+Y=n$, we have $E[XY] = E[X(n-X)] = nE[X]-E[X^2] =$

$$= nE[X] - E[X]^2 - \mathrm{Var}[X] = n\cdot np - (np)^2 - np(1-p) =$$

$$= n^2(p-p^2) - n(p-p^2) = n(n-1)p(1-p)$$

$$\mathrm{Cov}[X,Y] = E[XY]-E[X]E[Y] = n(n-1)p(1-p) - np\cdot n(1-p) =$$

$$= p(1-p)\left[n^2-n-n^2\right] = -np(1-p)$$

<u>Another method</u>: $X = X_1+\dots+X_n$, $Y = Y_1+\dots+Y_n$, where

$X_k = \begin{cases} 1, & k\text{ th experiment is a success} \\ 0, & \text{otherwise} \end{cases}$, $Y_k = 1-X_k$

Then $\mathrm{Cov}(X,Y) = \mathrm{Cov}\left(\sum_{k=1}^n X_k, \sum_{k=1}^n Y_k\right) \underset{\text{indep}}{=} \sum_{k=1}^n \mathrm{Cov}(X_k, Y_k) =$

$$= n\cdot \mathrm{Cov}(X_k, 1-X_k) = -n\,\mathrm{Var}(X_k) = -np(1-p)$$