1. Let $X$, $Y$, and $Z$ be three independent random variables such that
$E(X) = E(Y) = E(Z) = -1$ and $Var(X) = Var(Y) = Var(Z) = 1$.
Calculate $E[(X^2 + Y^2 + Z^2)]$.

\[
E[(X^2 + Y^2 + Z^2)] = E[X^2 + Y^2 + Z^2] = E[X^2] + E(Y^2) + 2E[XY] + 2E[XZ] + 2E[YZ]
\]

\[
= E[X^2] + E(Y^2) + 2E[XY] + 2E[XZ] + 2E[YZ] = 2.2 + 10.2(-1)(-1) + 25.2.2 = 124
\]

Since $E[X^2] = Var(X) + E(X)^2 = 1 + 1 = 2$, some $\beta$, $E[Y^2]$ and $E[Z^2]$.

2. In $n$ independent Bernoulli trials, each with probability of success $p$, let $X$ be the number of successes and $Y$ be the number of failures.
Calculate $E(XY)$ and $Cov(X, Y)$.

Since $X + Y = n$, we have

\[
E(XY) = E(X(Y - X)) = nE(Y) - E(X^2) = nE[Y] - E(X^2)
\]

\[
= nE(Y) - E(Y)^2 - Var(Y) = n \cdot np - (np)^2 - np(1-p) = np(n-1)p(1-p)
\]

\[
Cov(X, Y) = E(XY) - E(X)E(Y) = n(\cdot p)(1-p) - np(n-1)p(1-p) = p(1-p)[n^2 - n - n^2] = -np(1-p)
\]

Another method: $X = X_1 + \ldots + X_n$, $Y = \bar{Y} + \ldots + \bar{Y}$, where

\[
X_k = \begin{cases} 1, & \text{if experiment is a success} \\ 0, & \text{otherwise} \end{cases}, \quad Y_k = 1 - X_k
\]

Then

\[
Cov(X, Y) = Cov(\sum_{k=1}^{n} X_k, \sum_{k=1}^{n} Y_k) = \sum_{k=1}^{n} \text{Cov}(X_k, Y_k) = n \cdot Cov(X_k, 1-X_k) = -n \cdot Var(X_k) = -np(1-p)
\]