

LOCAL RIGIDITY AND IMPLICIT FUNCTION THEOREMS II: WEIL'S WORK.

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This talk is expository and concerns some old work of Andre Weil. Though the subject is closely related to my talk in the Fellowship of the Ring seminar, the talks will be completely independent.

In 1964, Weil showed that a homomorphism π from a finitely generated group Γ to a Lie group G is *locally rigid* whenever $H^1(\Gamma, \mathfrak{g}) = 0$. Here π is locally rigid if any nearby homomorphism is conjugate to π by a small element of G , and \mathfrak{g} is the Lie algebra of G . The first half to two thirds of the talk will concern Weil's proof of this fact.

In the remaining time, I will discuss applications of this theorem. If Γ is a cocompact lattice in G , Weil proved that $H^1(\Gamma, G) = 0$. This implies local rigidity of the defining representation of cocompact lattices. I will describe briefly how Weil used his result to prove that any cocompact lattice in say $SL(n, \mathbb{R})$ for $n > 2$ is in fact contained in $SL(n, \overline{\mathbb{Q}})$. This is a partial result towards Margulis' famous arithmeticity theorem, which was proven roughly ten years later.

If time permits I will briefly discuss the proof that $H^1(\Gamma, G) = 0$ when Γ is a cocompact lattice in G . The full proof involves some Hodge theory, to identify the cohomology group with the space of harmonic forms, which I will not explain. I will explain what harmonic forms are and what properties of Laplacian's are used to deduce vanishing.