On the equivariant diffeomorphisms of a smooth G-manifold and its applications

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Let M be a smooth connected manifold. We denote by $\mathcal{D}(\mathcal{M})$ the group of diffeomorphisms of M which are isotopic to the identity through isotopies with compact support. Hermann and Thurston proved that the group $\mathcal{D}(\mathcal{M})$ is perfect. The result is relevant to the foliation theory. There are many results in this direction on the group of diffeomorphisms of M.

In this talk we shall conider the case when M has a smooth G-action. Let $\mathcal{D}_G(M)$ denote the group of equivariant diffeomorphisms of M which are isotopic to the identity through equivarainat isotopies with compact support. Then $\mathcal{D}_G(M)$ is not perfect in general, and we calculate the first homology group $H_1(\mathcal{D}_G(M))$.

First, I will explain our recent results when M has a few orbit types. Secondly we calculate the first homology group $H_1(\mathcal{D}_G(V))$ for any representation space V of a finite group G. Using this result we can calculate $H_1(\mathcal{D}(\mathcal{M}))$ when M is a smooth orbifold and $H_1(\mathcal{D}_G(M))$ for a smooth G-manifold with finite group G. Thirdly, we apply the above results to the cases where M is a smooth G-manifold with G a compact Lie group or a foliated manifold, and also apply to the modular group.

We can calculate the first homology group of the above correponding automorphisms in Lipschitz category.

Most of those results are joint work with K. Fukui.