

# THE SHRINKING TARGET PROPERTIES

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ABSTRACT. We define and discuss a suite of shrinking target properties that some measure preserving dynamical systems of metric spaces have. Consider a sequence of open balls  $\{B(x, r_n)\}$  and a sequence of measure preserving self-maps  $\{f_n : M \rightarrow M\}$ . If the sum of the measures of these open balls diverges, what can we say about the limsup set,  $\mu(\{y \in M \mid f_n(y) \in B(x, r_n) \text{ for infinitely many } n\})$ ?

For finite products of expanding circle maps, every point will have limsup sets of full measure. We say that finite products of expanding circle maps have the shrinking target property (STP).

However, toral translations do not have STP. Can we weaken STP so that toral translations will satisfy some weaker version of STP? The answer, for a particular weakening called the monotone shrinking target property (MSTP), is sometimes. There is an interesting bifurcation that occurs between toral translations that have MSTP and those that do not. This bifurcation depends on the Diophantine properties of certain matrices, and, hence, belongs to the intersection of dynamical systems and number theory.

Finally, if time permits, we will mention that all dynamical systems of circle homeomorphisms with non-atomic Borel measure do not have STP.