UNIVERSALLY OPTIMAL DISTRIBUTIONS OF POINTS ON SPHERES

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Abstract. Among the various characterizations of “dense” sphere packings in Euclidean space, or “dense” spherical codes, we may consider configurations of points which minimize potential energy for a suitable repulsive potential. For instance, we can try to minimize the function $f(C,k) = \sum_{x \neq y \in C} d(x,y)^{-k}$, for $C \subset S^{n-1} \subset \mathbb{R}^n$ a spherical code of fixed size $N$ (here $d(x,y)$ is the usual Euclidean distance between $x$ and $y$). We find a large class of spherical codes which are optimal for every positive value of $k$ (and indeed, for a larger class of potentials), and show their uniqueness in many cases. Our techniques involve linear programming bounds and build on those of Kolushov and Yudin and of Andreev. We also conjecture that $A_2, E_8$ and the Leech lattice are optimal for a large class of potentials, among all periodic configurations of the same density. This is joint work with Henry Cohn.