Due Wednesday, February 4.

Be sure to write clearly, using complete sentences. Do not use abbreviations like s.t., w/, w/o, b/c, c/o, etc. In all problems you must prove that your answer is correct, even if the problem does not explicitly ask you to do so.

1. Problem 1.45 Page 24. For each definition, check whether or not the function is well-defined.

2. Problem 1.49 Page 24. Recall that f is bounded if there is a real number M such that $|f(x)| \leq M$ for all $x \in \mathbb{R}$. In part (c) consider the possibility that the values of $f + g$ are small while the values of f and g are large and cancel each other when you add them. Something similar happens in (d).

3. Let P be the set of positive numbers, and B be the set of numbers x such that $0 < x < 1$. Define $f : P \rightarrow B$, $f(x) = 1/(1 + x^2)$. Prove that f is a bijection by using the Bijectivity Test. (You should find the inverse of f first.)

4. Problem 2.1 Page 44.

5. *Fractional powers.* For positive $a \in \mathbb{R}$, we define the symbol $a^{2/3}$ to be the number $\sqrt[3]{a^2}$. Prove that this number is equal to $(\sqrt[3]{a})^2$. (Hint: By the Cubic Root Theorem, this number is the *unique* solution to the equation $x^3 = a^2$.) Let p, q to be positive integers such that $p/q = 2/3$. Define the symbol $a^{p/q}$ to be the number $\sqrt[q]{a^p}$. Prove that this number is also equal to $\sqrt[3]{a^2}$.

6. *The Fractional Power Law.* Do this one for fun. Do not hand in your proof but feel free to discuss it with me. Let p, q, r, s be positive integers. For positive $a \in \mathbb{R}$, we define the symbol $a^{p/q}$ to be the number $\sqrt[q]{a^p}$. Prove that $a^{p/q}a^{r/s} = a^{p/q+r/s}$ and that $(a^{p/q})^{r/s} = a^{(pr)/(qs)}$. (Hint: Consider the equations $x^{qs} = a^{ps+qr}$ and $x^{qs} = a^{pr}$ respectively and use the nth Root Theorem.) You can also prove that if $p/q = r/s$, then $\sqrt[q]{a^p} = \sqrt[s]{a^r}$ by a similar consideration.