

Due Wednesday, March 18.

Be sure to write clearly, using complete sentences. Do not use abbreviations like s.t., w/o, b/c, c/o, etc. In all problems you must prove that your answer is correct, even if the problem does not explicitly ask you to do so.

1. Problem 8 Page 95. Expand the formulas for $f \circ g$ and $g \circ f$.
2. Problem 11 Page 95. Explain this in mathematical language in no more than 5 lines.
3. We say that a function $g : \mathbb{R} \rightarrow \mathbb{R}$ is increasing when the following holds: for $x_1, x_2 \in \mathbb{R}$, if $x_1 > x_2$ then $g(x_1) > g(x_2)$. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function which is bijective and increasing. In no more than 20 lines, prove that the inverse function $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$ is increasing. (Try to use a contrapositive argument. For $y_1, y_2 \in \mathbb{R}$, assume that $f^{-1}(y_1) \leq f^{-1}(y_2)$, and then argue that $y_1 \leq y_2$.)
4. Let $m, n \in \mathbb{N}$, and A, B be disjoint finite sets. Let $f : [m] \rightarrow A$ and $g : [n] \rightarrow B$ be bijections. In no more than 20 lines, give a bijection $h : [m+n] \rightarrow A \cup B$; you must prove that the h you give, is a bijection. (Experiment with the case $m = 2, n = 3$.)
5. Let A and B be finite sets, whose cardinalities are $m, n \in \mathbb{N}$ respectively. In no more than 10 lines, prove that the cardinality of $A \cup B$ does not exceed $m + n$. (Note that $A \cup B = A \cup (B - A)$; A and $B - A$ are disjoint; $B - A \subset B$.)
6. Problem 1 Page 95.