Math 23b **Homework 6** **Spring, 2009**

Due Monday, March 30.

Be sure to write clearly, using complete sentences. Do not use abbreviations like s.t., w/, w/o, b/c, c/o, etc. In all problems you must prove that your answer is correct, even if the problem does not explicitly ask you to do so.

1. Compute the following in base 3, and verify your answers by redoing the computations in decimal representations and convert the results back to base 3:

$$12012 - 2221 =, \qquad 12012 \times 2221 = .$$

2. Suppose that you have a balance and weights of 1, 3, 9 and 27 oz . How can you balance a weight of 17 oz? You need to find $b_0, b_1, b_2, b_3 \in \{-1, 0, 1\}$ such that $17 = \sum_{i=0}^{3} b_i 3^i$. Note that $40 = \sum_{i=0}^{3} 3^i$. To find the b_i, it is easier to find the base 3 representation of $17 + 40$ first.

3. Let $A_1, A_2, A_3, ...$ be a list of countable sets. In no more than 1 page, prove that their union $A = \cup_{i \in \mathbb{N}} A_i$ is countable. Note that $x \in A$ if and only if $x \in A_i$ for some $i \in \mathbb{N}$. (List the members of the union in an infinite array, and use it to define a surjection $f : \mathbb{N} \to A$. Then apply a result we proved in class.)

4. Problem 4 Page 268. Justify your answer in no more than 10 lines.

5. Problem 8 Page 268. Recall that S bounded means that there is a number $M > 0$ such that $|x| \leq M$ for all $x \in S$. Justify your answer in no more than 10 lines.

6. Let S be a nonempty set of real numbers which has a lower bound. Let $-S$ denote $\{-x : x \in S\}$. In no more than 10 lines, show that $-S$ has an upper bound. By the Least Upper Bound Property, $-S$ has a least upper bound α. In no more than 20 lines, show that $-\alpha$ is the greatest lower bound of S.