Math 23b	Homework 7	Spring, 2009

Due Wednesday, April 8.

Be sure to write clearly, using complete sentences. Do not use abbreviations like s.t., w/, w/o, b/c, c/o, etc. In all problems you must prove that your answer is correct, even if the problem does not explicitly ask you to do so. In addition, one third of the grade on each exercise will be determined by the presentation of your argument. Even if your answer is in the end correct, you will lose points if there are irrelevant, extraneous or incorrect statements in your argument. There will be no revision for this homework. Since there is restriction on how much you can write in each problem, you should consider carefully what is essential to include, before writing your final answers.

1. Let $y > x$.

   (a) In 1 line, prove that there is a positive integer n such that $n(y - x) > 1$. (Use the Archimedean property.)

   (b) In no more than 10 lines, prove that there is an integer m such that $m > nx \geq m - 1$. (Consider the case $nx > 0$ and $nx \leq 0$ separately. Use the Archimedean property to find one m, and then use the Well Ordering property to find the smallest m.)

   (c) In no more than 5 lines, prove that

   $$nx < m < ny.$$

   (Combine (a) and (b).)

   (d) In no more than 3 lines, show that between any two distinct real numbers, there is a rational number.

2. Problem 11 Page 268. Do each part in no more than 20 lines. One of the statements is false. Think about the sequences $1/n$ and $-1/n$.

3. Problems 22b Page 269. Do this in no more than 30 lines. Solve the inequality that defines the set first.

4. Problem 23 Page 269. Do this in no more than 30 lines. You can use Proposition 13.15 and the fact that $lim\ (a_n + b_n) = lim\ a_n + lim\ b_n$ when all three limits exist.

5. Problem 28 Page 269. Do this in no more than 20 lines. (Hint: Can we assume that $\langle y \rangle$ has a limit?)

6. Problem 29 Page 270. Do this in no more than 30 lines. First show that $x_{n+1} \leq x_n$ for all n, so that you can apply the Monotone Convergence Theorem. To prove that the limit is $1/2$, write $x_n - 1/2$ as a fraction . You should see that $x_n - 1/2$ is nonnegative and less than $1/(4n)$. Then apply Proposition 3.12.