

Due Wednesday, March 18

1. Problems 35, 36, 37.
2. Problem 3 page 299.
3. Let  $f = 8x^4 - 8x^2 + x + 1$ . Check that  $-1, \frac{1}{2}$  are roots of  $f$ . Find the remaining two roots of  $f$ .
4. Put  $\theta = \frac{\pi}{5}$ . Prove that  $\cos \theta = \frac{1+\sqrt{5}}{4}$ . (Hint: Use  $\cos \frac{1}{2}\theta = \sin 2\theta$  to find a polynomial relation for  $\cos \theta$ . You should recognize that relation.) Conclude that the angle  $\theta$  is constructible. That is, there are two constructible lines that intersect at that angle. Show that the regular 10-gon, pentagon, and 20-gon are all constructible. That is, their edges are constructible lines.
5. Let  $L$  be the smallest subfield of  $\mathbb{C}$ , such that every degree 2 polynomial  $f \in L[x]$  has a root in  $L$ . Prove that  $L = K$ , the field of constructible complex numbers.