Math 30b	**Homework 7**	**Spring, 2009**

Due Wednesday, April 29

1. (a) By drawing a picture using straightedge and compass, describe a procedure for adding two constructible angles θ_1, θ_2. In other words, assuming that $e^{i\theta_1}, e^{i\theta_2}$ are constructible numbers, draw a picture to show how to construct the number $e^{i(\theta_1+\theta_2)}$.
   (b) Now suppose you have already constructed the angles $\frac{2\pi}{p}$ and $\frac{2\pi}{q}$, where p, q are two distinct Fermat primes. In less than a page, describe an explicit procedure, for constructing the angle $\frac{2\pi}{pq}$.

2. (a) Make a complete list of all integers n with $3 \le n \le 100$, such that the regular n-gon is constructible.
   (b) For the each n in your list, find the degree of the cyclotomic field $\mathbb{Q}(e^{\frac{2\pi i}{n}})$ over $\mathbb{Q}$.

3. Let $g = x^4 - 4x^2 - 1$, and K be the splitting field of g over $\mathbb{Q}$.
   (a) Describe it as a subfield of $\mathbb{C}$, and find $[K : \mathbb{Q}]$. What is the order of $Gal\ K/\mathbb{Q}$?
   (b) Argue that K is a Galois extension of $\mathbb{Q}$.
   (c) Find all elements in $Gal\ K/\mathbb{Q}$.
   (d) Work out the group diagram and the field diagram for $K/\mathbb{Q}$.

4. Problem 6 Page 469.

5. Problem 11 Page 470.

6. Problem 8 Page 475.

7. Show that the general degree 3 equation is solvable by radicals. (Show that the Galois group of the equation is S_3. Then show that S_3 is solvable.)