

# TEACHING PORTFOLIO

MARIO O. BOURGOIN

## CONTENTS

1. My Training at Brandeis	2
2. Syllabus for Introduction to Statistics	3
3. Hands-On Handout For Abstract Algebra	6
4. Error Bounds for Numerical Integration	9
5. A Quiz for Differential Calculus	11
6. Teaching The Fundamental Theorem of Calculus	13
7. Student Comments	15

## 1. MY TRAINING AT BRANDEIS

The Brandeis teaching program aims to develop the graduate students into high-quality teachers. It was founded in 1992 by Professor Susan Parker, and is a peer- and faculty-supported program. It consists of two semesters of one-on-one tutoring of undergraduates who are taking calculus, an apprenticeship with a current calculus teacher, a review with the mentor and faculty of classes taught as an apprentice, comprehensive end-of-year student evaluations, and yearly faculty review. The program is focused on teaching the basic service classes of the department: pre-calculus, differential calculus, and integral calculus. Recently, the program has added the teaching of multi-variable calculus and of linear algebra in combination with faculty.

Naturally, each of the courses's sections have the same syllabus and exams. But instead of the traditional lecture and recitation structure for such classes, all graduate student teachers are instructors for their own sections. In each section, the instructor sets an individual pace of instruction, assigns homework, designs quizzes, and is responsible for dealing with student issues, with the close support and advice of the course coordinator. Each year, an instructor's class is visited by one of the senior faculty to provide the instructor with feedback as to pace, content, methods, and relationship with students.

This program was helpful to me in developing my teaching philosophy in many ways. The independence of the sections allowed me to focus on my teaching style and its impact on my students. The regular contact with many teachers going through the same experience, my graduate student peers, provided me with many different models of how my course could be taught. The discussion of teaching approaches with my peers provided me with unusual ideas, some of which I was able to apply in my courses. One example is providing my class with worksheets that include all of the examples I work through in class but set in problem form, and also including other problems, to encourage the more self-directed students to explore the mathematics on their own.

## 2. SYLLABUS FOR INTRODUCTION TO STATISTICS

The next two pages include a syllabus for a statistics course intended to be a terminal course for students with no calculus background. This document is typical of the kind of syllabus I like to provide. It is spare and short, and provides a quick index to all course-related mechanics my students might need: the text and how much I intend to cover, the policies I will follow, sources of assistance such as additional materials or my office hours, and a calendar reminding the students of the important course dates. It reflects my expectation that my students are intelligent adults who can choose to take advantage of learning opportunities they have access to, be it personal contact, engagement in working out problems, extra practice sessions, etc.

# MATH 8A: INTRODUCTION TO PROBABILITY AND STATISTICS, Fall 2008

Instructor: Mario Bourgoïn, Goldsmith 204, x63063, mob@brandeis.edu

**TEXT.** *Introduction to the Practice of Statistics*, D. S. Moore and G. P. McCabe, 5<sup>th</sup> edition.

**SYLLABUS.** We will cover Chapters 1–9 and 12, and additional material as time permits.

**GRADES.** Your grade in the course will be based on the following:

**25% Midterm, Monday, October 27, 7–9 PM, Gerstenzang 122**

**30% Final Exam, Monday, December 15, 1:30 PM–4:30 PM**

**25% Group Project, December 1 and 3, in class**

**20% Homework and Class Participation**

## COURSE POLICIES.

**EXAMS.** Please note that the midterm exam is in the evening. If you have a genuine and irremovable schedule conflict (such as a class, lab, or another exam) with the MATH 8A midterm exam, inform me at least two weeks before the exam. If the conflict can't be resolved, we will offer you a make-up exam. *You must obtain permission from me to miss an exam BEFORE the exam is held.* Rescheduling final exams can only be done in exceptional circumstances, and *under no circumstances will final exams be given early.*

**GROUP PROJECT.** All students are expected to work with one or two other students on a group project. The results of the project will be a paper and an in-class PowerPoint presentation at the end of the semester. *You must obtain permission from me to miss your presentation BEFORE the presentation days.* Any research that involves human subjects must conform to the guidelines set forth by the Brandeis Committee for Protection of Human Subjects. The project will be discussed in greater detail in class.

**ASSIGNMENTS.** Homework assignments will be collected regularly. *No late assignments will be accepted*, but your three lowest homework grades will be dropped. We encourage you to discuss homework problems with your classmates, but *you should write the solutions up on your own.* We will usually not have time to discuss homework questions in class. If you have questions about homework problems, you should go to my office hours.

**CLASS PARTICIPATION.** Prompt and regular class attendance is expected – more than two unexcused absences will result in a lower grade for participation. *Two late arrivals (more than 5 minutes late) will count as one absence.* Beyond participation in discussions and group projects, students will be expected to read the Boston Globe or the New York Times regularly. At the beginning of each class a student will report on "Statistics in the News," a description, interpretation, and criticism of graphs and statistics found in the news.

**CALCULATORS.** You should have a statistics calculator with capabilities comparable to the TI-30X II S. You do not need a graphing calculator. *Calculators are required during exams*, and they will be used in class.

**STUDENTS WITH DISABILITIES.** If you are a student who needs academic accommodations because of a documented disability you should contact me and present your letter of accommodation without delay. If you have questions about documenting a disability or requesting academic accommodations you should contact Beth Rodgers-Kay in Undergraduate Academic Affairs at 6-3470. Letters of accommodation should be presented at the start of the semester to ensure provision of accommodations. *Accommodations cannot be granted retroactively.*

*ACADEMIC INTEGRITY.* You are expected to follow the University's policy on academic integrity (see <http://www.brandeis.edu/studentlife/sdc/ai>). In particular, you are expected to be familiar with Section 4 of the Rights and Responsibilities handbook. If you have any questions about how these policies apply to your conduct in this course, please ask. *Instances of alleged dishonesty will be forwarded to the Office of Student Life for possible referral to the Student Judicial System.*

### COURSE ASSISTANCE.

*LATTE.* All course materials for MATH 8A will be available on line on LATTE. Log in to LATTE at <http://moodle.brandeis.edu> using your Unet username and password.

*OFFICE HOURS.* You are encouraged to use my office hours whenever you have questions about the course material. If you can't attend my office hours, make an appointment with me to meet at another time.

*EMAIL.* Email is a good way of getting in touch with me. I try to answer all email promptly, although it will sometimes take me up to 24 hours before I can get to your email.

### SEMESTER CALENDAR.

Monday	Tuesday	Wednesday	Thursday	Friday
			<b>Aug 28 First</b>	29
<b>Sep 1 Holiday</b>	2	3	4	5
8	9	10	<b>11 Last Add</b>	12
15	16	17	18	19
22	23	24	25	26
29 Tuesday	<b>30 Holiday</b>	<b>Oct 1 Holiday</b>	2	3
6	7	8	<b>9 Holiday</b>	<b>10 Holiday</b>
13	<b>14 Holiday</b>	15	<b>16 1<sup>st</sup> Drop</b>	17
20	<b>21 Holiday</b>	22	23	24
<b>27 Midterm</b>	28	29	30	31
<b>Nov 3</b>	4	5	6	7
10	11	12	13	<b>14 2<sup>nd</sup> Drop</b>
17	18	19	20	21
24	25	26	<b>27 Holiday</b>	<b>28 Holiday</b>
<b>Dec 1 Present</b>	2	<b>3 Present</b>	4	5
8	9 Last	10	11	12
<b>15 Final</b>				

### 3. HANDS-ON HANDOUT FOR ABSTRACT ALGEBRA

In 2005, I ran a course in abstract algebra for a group of students with a wide-range of preparation, from no algebra to calculus. I chose to make the course exploratory using hands-on materials. I led discussions of the students' explorations, and I gave short lectures to introduce new ideas. The handout in the next two pages is the introductory handout for the course. It informally presents the concept of planar symmetries, leads the student into creating an equilateral triangle out of a sheet of paper, and invites her to partner with another student to explore the possible ways an equilateral triangle can be laid on top of another. This raises many issues for discussion beyond symmetry, including the problem of notation.

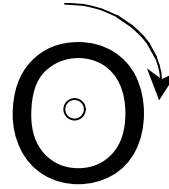
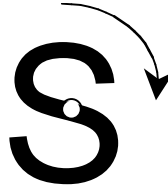
# Group Theory

## 1. Symmetry

An image has symmetry with respect to a transformation if it is the same after the transformation. Here are two kinds of symmetries of letter shapes:



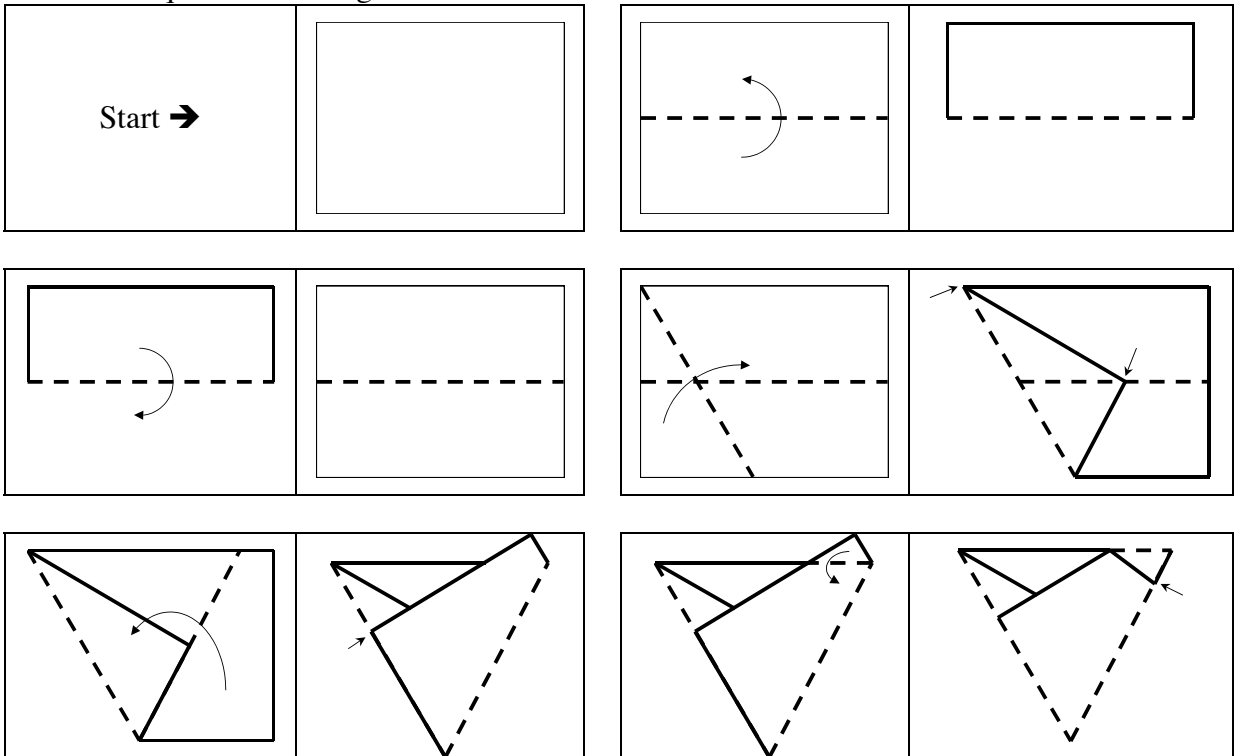
Reflection  
Symmetry



Rotation  
Symmetry

**What are the possible symmetries of an equilateral triangle?**

Make an Equilateral Triangle Out of a Sheet:



***An exploration:***

- Find a partner.
- Label the vertices of your triangles in the same way.
- Record the all the different ways of putting one triangle on top of the other one.

***Questions:***

1. How many different ways did you find?
2. Was there one way that put the all labels of one triangle on the corresponding labels of the other triangle?

***Notes***

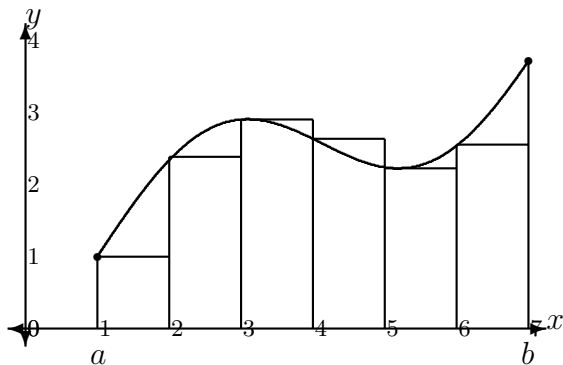
#### 4. ERROR BOUNDS FOR NUMERICAL INTEGRATION

I recently introduced the next document in my undergraduate integration class to help interested students understand how error bound formulas for midpoint and trapezoid approximation that we accept on faith might have been derived. I give them this handout after they have had many weeks of experience with the left-hand and right-hand approximations to the area under a curve, and intuitively know many of the facts that are shown. I have also introduced the midpoint and trapezoid error bound formulas, and have motivated their form and properties using diagrams. However, these error bounds formulas are accepted as the product of numerical analysis, without any derivation.

For left- and right-hand approximation error bounds, I begin with a concrete situation they already understand, and progressively move towards an abstract formula for the left-hand error-bound formula that is similar in form to those for the midpoint and trapezoid error bound. I continue by inviting the student to carry out this derivation to find the error bound formula for the right-hand approximation. I usually stop here during the class, but I invite the students to explore the rest of the sheet on their own. There, more advanced students are introduced to the idea of a sharp error bound, and that an error might be maximally far from the error bound.

## MATH 10B HANDOUT – ERROR BOUNDS FOR $L_n$ AND $R_n$

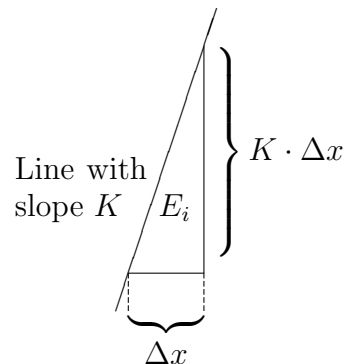
Suppose we have a continuous function  $f(x)$  over  $[1, 7]$  whose integral we approximate with  $L_6$ :



Where the function is increasing, a rectangle underestimates the area, and where it is decreasing, it overestimates the area. The sum of these errors is  $E_L$ .

Suppose we know that  $f$ 's largest slope on  $[1, 7]$  is 1.5 in absolute value terms. If  $f$  had that slope over every subinterval, the error would be the same for every rectangle, namely  $\frac{1 \cdot 1.5}{2}$ , and the sum of these errors over the six rectangles would be  $6 \cdot \frac{1 \cdot 1.5}{2} = 4.5$ . Since 1.5 is the largest slope of  $f$  over  $[1, 7]$ , this error must be larger than what we have when we use the actual curve:  $|E_L| \leq 4.5$ .

Let's generalize this to  $L_n$ , the left-hand approximation with  $n$  intervals of  $\int_a^b f dx$ . We let  $K \geq |f'(x)|$  on  $a \leq x \leq b$ , a number bigger than our largest slope for  $f$  on the interval. If  $f$  had that slope at every point, the error would be the same for every rectangle, namely  $E_i = \frac{\Delta x \cdot (K \cdot \Delta x)}{2}$  as is shown on the right. We add up all of these  $n$  errors and use  $\Delta x = \frac{b-a}{n}$ .



$$\sum_{i=1}^n E_i = \sum_{i=1}^n \frac{K \cdot \Delta x^2}{2} = n \cdot \frac{K \cdot \Delta x^2}{2} = n \cdot \frac{K(b-a)^2}{2n^2} = \frac{K(b-a)^2}{2n}.$$

This number is at least as large as the error of the left-hand rule with  $n$  intervals using the actual function:  $|E_L| \leq \frac{K(b-a)^2}{2n}$ .

**Exercise.** Use the previous reasoning to show that  $|E_R| \leq \frac{K(b-a)^2}{2n}$ .

**Definition.** An error bound formula is called *sharp* if there is a function  $f$  for which it calculates the actual error.

The error bound formula for  $L_n$  is sharp because it calculates the error  $E_L$  for any line  $f(x) = mx + b$  with positive slope  $m$ .

**Exercise.** Check that  $E_L = \frac{K(b-a)^2}{2n}$  for  $\int_0^5 (3x+2) dx$  for  $n = 1, 2$ , and 5.

For other functions, the error  $E_L$  can be much less than the error bound.

**Exercise.** Check that if  $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x \leq 2 \end{cases}$ , estimating  $\int_0^2 f(x) dx$  with  $L_n$ ,  $n = 2, 4, 6, \dots$ ,

has  $E_L = 0$ .

**Exercise.** Is the error bound formula for  $R_n$  sharp? Explain your answer.

## 5. A QUIZ FOR DIFFERENTIAL CALCULUS

I believe in giving my students frequent quizzes aimed at the center of a topic we've recently covered. This gets them accustomed to answering exam questions, and give them a sense of what the more ordinary questions might look like. It also provides me with an objective measure of how well my students understood the concepts I introduced. I use this to adjust the level of detail I give my students as I teach later topics that depend on the one covered by the quiz. Typically, a quiz includes a single question that, if possible, touches on all the main aspects of a topic. The quiz on the next page is given about a week after the students have handed in the homework on using the definition of a derivative. The quiz asks the students to use this definition in a relatively straightforward way, and then has them apply it to finding the tangent line at a point to remind them of the way we came to the definition of the derivative.

1. Let  $f(x) = \sqrt{x+5}$ .
  - (a) Using the definition of the derivative, find  $f'(x)$ .

(b) Find the equation of the tangent line to the graph of  $f$  at  $x = 4$ .

## 6. TEACHING THE FUNDAMENTAL THEOREM OF CALCULUS An Example of The Evolution of My Thinking About Teaching

Teaching the Fundamental Theorem of Calculus (FTC) to undergraduates has special challenges. At first, my students have only worked with functions given as tables, graphs, and closed form expressions. Now they get introduced to a whole new way of writing down a function: as an integral. So I must help them understand how to work with this very important form: how to evaluate the function where possible, how to approximate its value when the integral has no closed form, how to find the function's direction, extrema, concavity, inflection points, behavior at infinity, etc.

This process begins with finding the derivative of an integral. In being introduced to the FTC, my students learn that  $\frac{d}{dx} \left( \int_a^x f(t) dt \right) = f(x)$ . While an application of this fact seems to us straightforward, the conceptual leap can be daunting for some students. Up to now, they've learned how to differentiate familiar closed forms, and how to solve indefinite integrals into closed forms. But now, this familiar ground is pulled from under their feet, and they have to really deal with integrals on the same footing as closed forms.

If making that conceptual leap isn't enough, my students are soon faced with integrals where the upper bound is an expression in  $x$ , and then they have to understand that such integrals express a composition of functions in order to find their derivatives. But what might seem at first a challenge made harder, is actually an opportunity in disguise since my students must now look at the integral in the context of something that is special about functions: they can be composed<sup>1</sup> to give new functions. My problem is how to best take advantage of this opportunity to help my students see that an integral can be used to define a function with which they can work.

**6.1. Thesis.** The original approach to exposing the power of integrals through composition came from making a more comprehensible version of an early example of using the FTC in the course text:

$$\text{Find } \frac{d}{dx} \int_1^{x^4} \sec t dt.$$

In finding the solution to this example, the course text spoke of the need to use the Chain Rule along with the FTC, and then showed how this might be done to find the solution. We felt that to reach more students, a more explicit approach was needed:

**Example.** Let  $f(x) = \int_1^{x^4} \sec t dt$ . Find  $f'(x)$ .

*Solution.* Let  $g(x) = \int_1^x \sec t dt$ . We know that  $g'(x) = \sec x$ . Notice that  $f(x) = g(x^4)$ . If we use the Chain Rule to compute  $f'(x)$ , we get  $f'(x) = g'(x^4) \cdot 4x^3$ . And since  $g'(x) = \sec x$ ,  $f'(x) = \sec(x^4) \cdot 4x^3$ .

We followed this by a number of examples that show the range of the phenomena, to motivate our students of the need to understand it.

Invariably, some students were puzzled by the presence of expressions in  $x$  as the bounds of the integral. And while they could readily see the mechanics of what we did, the fact we didn't explicitly use the original integral encouraged a sense of disconnection of the solution from the problem. Finally, some found cases where the lower bound was an expression or both bounds were expressions, particularly difficult.

**6.2. Antithesis.** Once, in a quiz problem about the FTC, I used an integrand whose anti-derivative was known to my students. One of them solved the problem this way:

1. Let  $f(x) = \int_1^{x^4} \cos t dt$ . Find  $f'(x)$ .

---

<sup>1</sup>For functions for which composition makes sense, of course.

$$\begin{aligned} \textit{Solution. } f(x) &= \sin t \Big|_1^{x^4} = \sin(x^4) - \sin(1). \\ f'(x) &= (\sin(x^4))' - (\sin(1))' = \cos(x^4) \cdot 4x^3 - 0 = \cos(x^4) \cdot 4x^3. \end{aligned}$$

I was struck by the simplicity of the solution, that it used the method of evaluation of definite integrals using anti-derivatives as our students had just been taught, that it solved the problem by explicitly using the original integral, and that it readily generalized to other cases. I decided to adapt this approach to my teaching of this example:

**Example.** Let  $f(x) = \int_1^{x^4} \sec t \, dt$ . Find  $f'(x)$ .

*Solution.* We don't yet know whether  $\sec t$  has an anti-derivative as a formula. If it does have an anti-derivative  $g(x)$ , that is if  $g'(x) = \sec x$ , then  $f(x) = g(x^4) - g(1)$ . And by the Chain Rule,  $f'(x) = (g(x^4))' - (g(1))' = g'(x^4) \cdot 4x^3 - 0 = \sec(x^4) \cdot 4x^3$ .

When I presented this to my students, some were confused by the lack of a definition of the anti-derivative  $g(x)$ , so I felt I had not improved the explanation enough.

**6.3. Synthesis.** It then occurred to me to combine the best aspects of these two approaches: explicit definition of  $g(x)$ , solution of the original integral, and then differentiation of  $f(x)$ :

**Example.** Let  $f(x) = \int_1^{x^4} \sec t \, dt$ . Find  $f'(x)$ .

*Solution.* We don't yet know whether  $\sec t$  has an anti-derivative as a formula. We do know from the FTC that  $g(x) = \int_1^x \sec t \, dt$  is an anti-derivative for  $\sec t$  because  $g'(x) = \sec x$ . Then  $f(x) = g(x^4) - g(1)$ , and by the Chain Rule,  $f'(x) = (g(x^4))' - (g(1))' = g'(x^4) \cdot 4x^3 - 0 = \sec(x^4) \cdot 4x^3$ .

Now, I have a complete explanation for my students that relies upon what they have just learned to show that the problem's integral really is a composition of functions.

## 7. STUDENT COMMENTS

Here is a selection of recent comments from my student's evaluations of my classes from the Fall of 2006 through the Spring of 2009.

- *Math 10b: Integral Calculus, Spring 2009*

- There is a fair amount of course work assigned throughout the semester. Though it's tough, the numerous quizzes and assignments sort of forced me to keep on top of the material and be caught up in class. It helped a lot because it reduced the amount of cramming I would have normally done for the upcoming finals.

- The professor showed a lot of interest in the subject, and I liked the fact that he was able to answer all of our questions for us, no matter how ridiculous they were.

- *Math 10a: Differential Calculus, Spring 2009*

- He made Math fun!

- His enthusiasm for the topic was the best part. He made things sound simple yet he explained small and almost silly things keeping in mind that for some of us math is not simple. He managed to do this without making the class dull and too easy.

- *Math 10b: Integral Calculus, Fall 2008*

- He's a really dedicated professor, that tries his best to make people understand Math, and he does it in such a way that you are never afraid to ask questions in class. He's an overall great professor.

- *Math 8a: Introduction to Statistics, Fall 2008*

- I thought the "Statistics in the News" presentations were a perfect way to get us to see the use of what we learned in the real world.

- The instructor should be nominated for a teaching award because he makes sure student understand the subject and are attentive. He help student engage in statistic by assigning them "statistic in the news" so student need to find how statistic is used in today's world.

- *Math 10a: Differential Calculus, Spring 2008*

- Prof. Bourgoin is an excellent professor. He is extremely engaging in his lectures, keeps the class environment light and fun, and it really shows that he cares about our learning.

- The semester-long final project was a great way to get a taste for statistics.

- Prof. Bourgoin (*sic.*) isn't the most organized prof. But he really wants the students to

do well, and enjoy the topic. HE is the perfect Prof to be teaching a course that most students take for a university requirement.

- *Math 40a: Mathematical Analysis I, Fall 2007*

- The textbook and professor's notes which he puts online are amazing. Lectures were helpful, but I was not always able to follow the proofs. The homeworks were valuable for learning and understanding the material. The professor was also available and very helpful outside of the class.

- I do feel professor Bourgoin should be nominated for a teaching award. He's a very good lecturer, who manages to include the class very well, and makes himself very available to help students understand the material and discuss topics in mathematics in general. He displays a great enthusiasm for the subject which he can't help but pass on to his students.

- *Math 10b: Integral Calculus, Fall 2007*

- This professor has a good knowledge and is trying to share his passion with the rest of the class and to be helpful. Merci beaucoup pour les cours!

- *Math 40b: Mathematical Analysis II, Spring 2007*

- The prof. was always willing to talk to you + answer questions about assignments.

- *Math 10a: Differential Calculus, Spring 2007*

- He has a great way of stimulating interest and providing great examples.

- *Math 37a: Ordinary Differential Equations, Fall 2006*

- Good use of technology. Very patient. Pace was excellent.

- Instructor was humorous and took the time to teach the concepts he thought were relevant.

- *Math 10b: Integral Calculus, Fall 2006*

- He does and (*sic.*) amazing job of getting his point across.

- This instructor is very organized, neat, and clear in explaining. He makes calculus more approachable + understandable.