NOTE: In this exam, you may use any of the following three summation formulae:

\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \left[ \frac{n(n+1)}{2} \right]^2
\]

Write your answers and show your work in your exam book. Put your name on your exam & turn it in with your exam book. NO CALCULATORS ALLOWED. SHOW ALL YOUR WORK FOR FULL CREDIT.

1. (10 points) Evaluate the following.
   
   (a) \( \arcsin\left(\frac{1}{2}\right) - \arctan(-1) \)
   
   (b) \( \cot\left(\sin^{-1}\left(-\frac{2}{3}\right)\right) \)

2. (6 points) The water in a water tank contains all sorts of sediment from rust to insects, dirt, algae and bits of washed in plant matter, so many water tanks have a filter to help remove this sediment. Suppose that the following measurements show the rate (in grains/hour) at which a filter is removing the sediment from a water tank. The measurements are taken at the beginning of each half hour.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate at which sediment is removed (gr/hr)</td>
<td>8.2</td>
<td>7.4</td>
<td>5.8</td>
<td>5.2</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Use four subintervals to find a lower estimate for the amount of sediment removed during the first two hours.

3. (8 points) Let \( f(x) = 4^{\arctan(2x)} \). Find \( f'(0) \).

4. (5 points) Write the sum \( \frac{1}{5} - \frac{1}{10} + \frac{1}{15} - \frac{1}{20} + \frac{1}{25} - \frac{1}{30} + \frac{1}{35} - \frac{1}{40} \) using sigma notation.

5. (6 points) Give an example of a continuous function \( f(x) \) that is concave down on \([0, 1]\) for which every left Riemann sum over-estimates the integral \( \int_{0}^{1} f(x) \, dx \).

   Note: For full credit, you must give the formula for a function \( f(x) \), but partial credit will be given for a graph of \( f(x) \).

6. (6 points) Suppose that \( P(x) \) is a polynomial function of degree \( n \). We can write \( P(x) \) using sigma notation as follows:

   \[
P(x) = \sum_{i=0}^{n} a_i x^i,
   \]

   where \( a_0, a_1, a_2, \ldots, a_n \) are real numbers (the coefficients of the polynomial). Write the derivative \( P'(x) \) using sigma notation. Hint: Remember that \( x^0 = 1 \).

OVER
7. (4 points) Find the domain and range of the function \( f(x) = 5\sin^{-1}x \). Write both your answers in interval notation.

8. (11 points) Use the definition of the definite integral as the limit of a Riemann sum to evaluate \( \int_1^3 (3x^2 + 2) \, dx \). No credit for using shortcuts/techniques from Sections 5.3.

9. (7 points) Consider the limit \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n} \tan^2\left(\frac{i}{n}\right) \).

   (a) Express this limit as a definite integral \( \int_a^b f(x) \, dx \) with \( a = 0 \).

   (b) Express the same limit as a definite integral \( \int_a^b f(x) \, dx \) with \( a = 3 \).

   **Note:** Do not evaluate the integrals you found.

10. (11 points) Compute the following integrals. You may not use the second Fundamental Theorem of Calculus (Section 5.3).

   (a) \( \int_{-11}^{-5} |10 + x| \, dx \)

   (b) \( \int_0^3 \sqrt{36 - 4x^2} \, dx \)

11. (10 points) In mathematics, the error function is a special function that occurs in probability, statistics, and partial differential equations describing diffusion. The error function is defined as follows:

\[
f(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt.
\]

Is \( f(x) \) concave up or concave down on \((0, +\infty)\)? Show all your work.

12. (5 points) Find a function \( F(x) \) such that \( F'(x) = \ln(\cos x + 3) \) and \( F(2\pi) = 0 \).

13. (12 points) Let \( f(x) \) be continuous functions and suppose that

\[
\int_0^2 f(x) \, dx = 5, \quad \int_0^6 f(x) \, dx = 2, \quad \int_1^5 f(x) \, dx = 7 \quad \text{and} \quad \int_5^6 f(x) \, dx = -3.
\]

Some of the integrals listed below can be evaluated using properties of integrals and some can’t. Compute the ones that can be evaluated, showing your work. If an integral can’t be evaluated you may simply write “can’t evaluate”.

   (a) \( \int_1^6 (2f(x) + 3) \, dx \)

   (b) \( \int_2^4 f(x - 2) \, dx \)

   (c) \( \int_5^6 |f(x) + 3| \, dx \)

   (d) \( \int_1^0 f(x) \, dx \)

14. (10 points) Let \( g(x) = \int_0^{\sin x} \left(4 + \sin(t^2)\right) \, dt \). Find the equation of the line tangent to the graph of \( g(x) \) at \( x = \pi \).

\( \overline{\text{OVER}} \)
15. (6 points) Determine whether each of the following statements is true or false, and then clearly and succinctly explain why. No credit for answers without an explanation.

**Note:** In mathematics, “True” means that the statement must **always** be true. “False” means that the statement may sometimes be false.

(a) Let \( F(x) = \int_{0}^{x} f(t) \, dt \). If \( F \) is increasing on \((0, +\infty)\),

then \( f \) is also increasing on \((0, +\infty)\).

(b) Let \( a \) and \( b \) be real numbers and let \( f \) be a continuous function. Then

\[
\frac{d}{dx} \left( \int_{a}^{b} f(t) \, dt \right) = f(x).
\]

16. (8 points) Find \( \lim_{x \to \pi} x^{2} \int_{x}^{\pi} \frac{\cos t}{t} \, dt \). Show all your work!