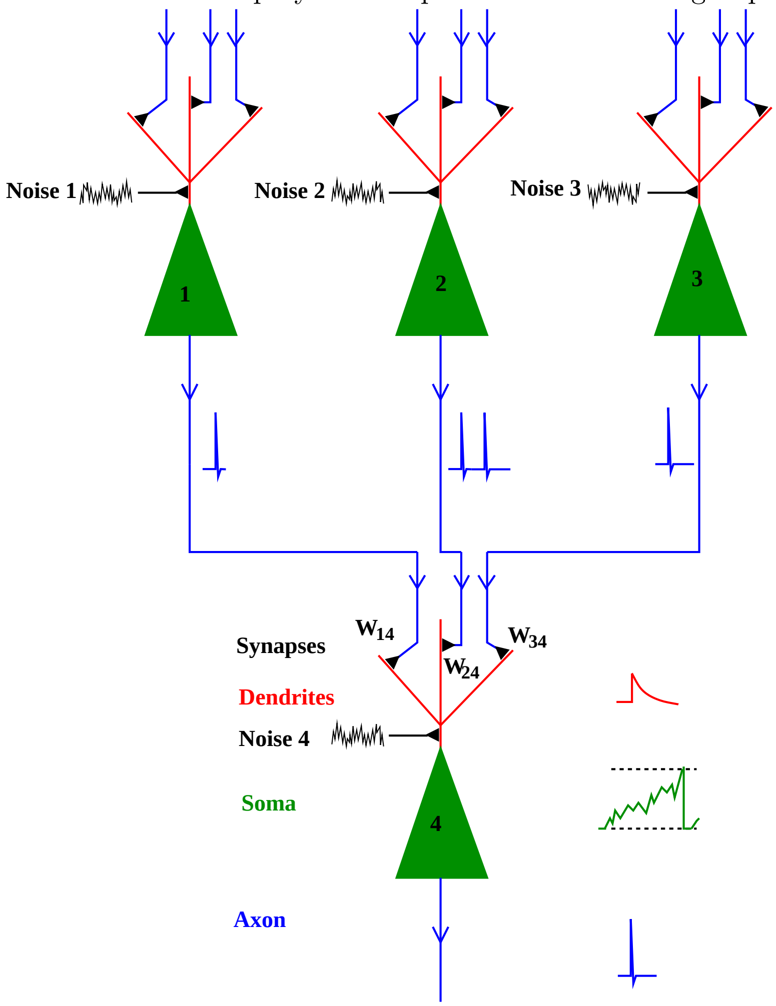
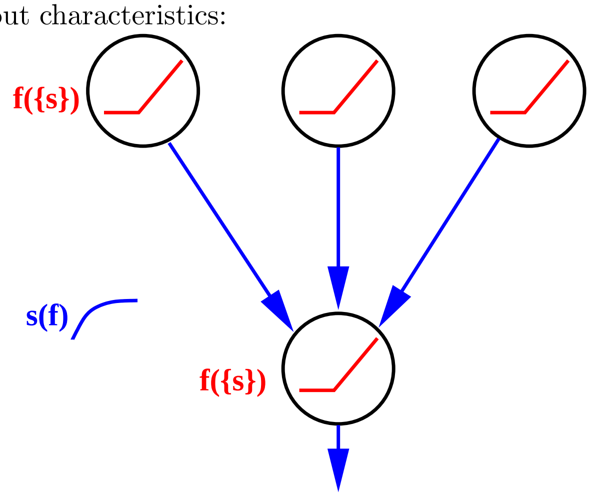
Firing Rate Models

Firing rate models depend on the assumption that the average firing response of a neuron to its inputs and the average effect of such firing on the inputs to any other neuron is enough to explain the important properties of a neuronal network.

Overall effect of a rate model is to simplify the computation such that a group of neurons:



gain simplified input-output characteristics:



The average responses can change dynamically, but in general will correspond to some sort of relaxation to a known steady state response.

The two quantities needed in the model are the inputs and the outputs of the neuron. Inputs can be currents, I, but here we use the synaptic gating variable, s (since activity of other neurons opens a fraction of channels and does not impart a known current).

Instead of a spike train, the measured output is the firing rate, r. In some models the CV is also calculated.

Possible assumptions:

Firing rate changes slowly in response to synaptic input; synaptic conductances change slowly in response to spike changes:

$$\tau_r \frac{dr}{dt} = -r + f(\{s\}) \qquad \tau_s \frac{ds}{dt} = -s + F(r) \tag{1}$$

or firing rate changes slowly but synapses change instantaneously:

$$\tau_r \frac{dr}{dt} = -r + f(\{s\}) \qquad s = F(r) \tag{2}$$

or firing rate responds instantaneously to synaptic input, but synaptic conductances change slowly:

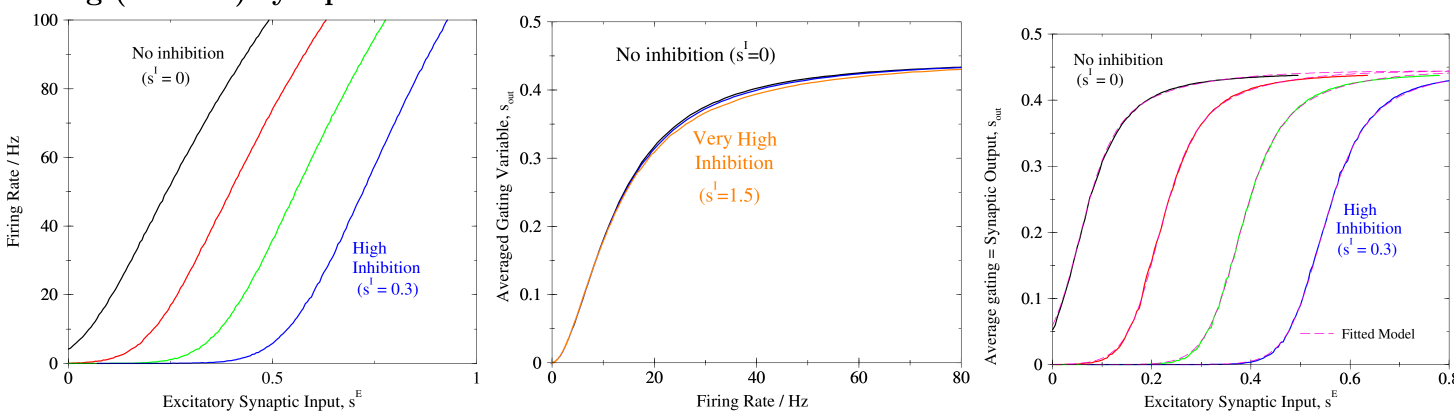
$$\tau_s \frac{ds}{dt} = -s + F(r) \qquad r = f(\{s\}). \tag{3}$$

I go with the final form. Note: the firing rate depends on the values of s at all synapses (hence $\{s\}$) but these can generally be split up into separate excitatory and inhibitory contributions and weighted by the relative strength of each synapse:

$$r_i = f(\{s_j\}) = f(S_i^E, S_i^I) = f\left(\sum_E s_j W_{ji}^E, \sum_I s_j W_{ji}^I\right) \tag{4}$$

where $\sum_E$ is the sum over all excitatory cells (labeled by j) and $\sum_I$ is the sum over all inhibitory cells. W_{ji}^E is the strength of an excitatory connection from cell j to cell i. W_{ji}^I is the strength of an inhibitory connection from cell j to cell i.

Figure showing simulated noisy leaky-integrate-and-fire neuron with output to a saturating (NMDA) synapse.



Notes: Firing rate models can include dynamical effects such as depression and facilitation (if ds/dt is calculated) and adaptation (if dr/dt is calculated).

Each term should represent a group of similar neurons, or population, so that spike times from members of a population are very close, reducing the noise in synaptic input to a connected population.

All that they lose is information on spike correlations at a finer timescale than any rate variations, determined by τ_r and/or τ_s.

Connections

Connections between neurons are given as synaptic weights, W_{ij} from neuron i to neuron j.

Four types of connectivity shown:

$$\begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 1 \\ 2 & 1 & 2 & 3 \\ 1 & 1 & 2 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 3 \\ 0 & 2 & 0 & 1 \end{pmatrix} \tag{5}$$