Bentley College
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PH 101: Problems of Philosophy
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NOTES ON LOGIC, PART II

I. THE CONCEPT OF VALIDITY

§ 1. Aspects of arguments. In § 22 of the first set of these notes, I observed that, in philosophy, arguments are evaluated chiefly with respect to their *cogency*, that is, the degree to which they provide us with compelling reason to accept their conclusions. The cogency of an argument, as I also said, depends on two things:

(i) the kind and degree of support that the premises, if true, provide for the conclusion, and
(ii) the rational acceptability of the premises themselves (i.e., the degree to which we have reason to accept them).

Logic is concerned with only the first of these aspects of arguments. It does not, in general, deal with the question whether any of the propositions in an argument are true, or whether we have any reason to believe that they are true, but only with the question whether the *inferences* made in the argument are legitimate; i.e., to what degree and in what way the premises of the argument, *if* true, give us reason to accept the conclusion.

§ 2. Validity and invalidity of arguments. The most important evaluative distinction among arguments that is made in logic is the distinction between *valid* and *invalid* arguments. A *valid* argument is an argument such that *if* all the premises are true then the conclusion *must* be true. Equivalently, a valid argument is an argument such that it is impossible for all the premises to be true and the conclusion not true. Another way of putting the point is to say that a valid argument is one in which the conclusion *necessarily follows from* the premises, or in which the truth of the premises *necessitates* or *guarantees* the truth of the conclusion. Any argument that is not valid is called an *invalid* argument.

§ 3. "Valid" and "invalid" are technical terms. Note that the terms "valid" and "invalid" as applied to arguments are *technical terms*: that is, they bear a special and well-defined meaning for the purposes of the discipline of logic. In the context of the evaluation of arguments, they should not be used or understood in any other sense. To call an argument "valid" is not the same as saying that it is a *good* or a *convincing* argument, and to call an argument "invalid" is not to say that the argument is a *bad* or an *unsuccessful* one. These points will be illustrated in the two sections that follow.

§ 4. Some examples of valid arguments. The following are examples of *valid* arguments, presented in standard form (on "standard form," see §§ 17–21 of the first set of notes on logic):

A. 1. All dogs can fly.
   2. All birds are dogs.
∴ 3. Therefore, all birds can fly. (From 1 and 2)

B. 1. If we have followed the directions, then this is Forest Street.
   2. This is not Forest Street.
∴ 3. So we have not followed the directions. (From 1 and 2)

Argument A may be called a *bad* argument because its premises are, obviously, false; but it is still a *valid* argument, according to the logical sense of the term, because it is such that it is impossible for the premises to be true without the conclusion also being true. If it were the case that all dogs can fly and that all birds are dogs, then it would necessarily also be the case (as in fact is the case) that all birds can fly.

As for argument B, you should be able to see that it is valid, even though you have absolutely no knowledge whether any step of it is true or false. You have no idea what people are being referred to as "we," where they are, what the directions said, and so forth; yet you should be able to see easily that *if* the premises (1 and 2) are both true, then the conclusion (3) is true also.

One point about the notion of validity that you should get quite clear is that the question whether an argument is valid or invalid is *independent of whether the premises are true or false*. It is also independent of whether the *conclusion* is true or false. Validity is a matter of the *relation* of the premises to the conclusion, namely whether the conclusion necessarily follows from the premises.

§ 5. Some examples of invalid arguments. The following are examples of *invalid* arguments:

C. 1. You need to take Philosophy 101 in order to graduate.
   2. This class is Philosophy 101.
∴ 3. Therefore, you need to take this class in order to graduate. (From 1 and 2)

D. 1. I called the store, and they said that they are open till 6:00.
   2. It's 5:00 now.
∴ 3. Therefore, the store will be open for another hour. (From 1 and 2)

You should be able to see that there is something wrong with argument C. It *looks* like a valid argument; but you know for a fact (I hope) that, even though steps 1 and 2 are both true, the conclusion is not true; therefore, the argument cannot be valid. (If the premises are all true and the conclusion false, then, plainly, it is not impossible for the premises to be all true and the conclusion false; thus the argument is not valid.) What is wrong with C is that it gains the appearance of validity only by exploiting an ambiguity of expression. The first step means that, in order to graduate, you need to take *some section or other* of Philosophy 101, while the second step means that this class is *a section* of Philosophy 101; so it does not follow that you need to take this class, that is, *this particular section* of Philosophy 101 in order to graduate. C is what is called a *fallacious* argument: one that has a concealed defect. Specifically, it commits what is called a *fallacy of equivocation* (trading on an ambiguity of expression). It is thus a *bad* argument. But what makes it *invalid* is simply the fact that the conclusion does not follow from the premises: they may be true while the conclusion is false.

Argument D is a different case. It is certainly not a bad argument: generally speaking, in any case in which the first two statements are true, the third statement is very likely to be true. But, *for all that the premises have told us,* it may be the case that the person who answered the phone was mistaken, or was, for heaven knows what reason, deliberately misleading you (perhaps he was not an employee but a mischievous customer); it may happen that, since you called, some dire emergency has occurred and caused the store to close early; and so on. There are innumerable conceivable ways in which it can happen that the premises are both true while the conclusion is false. The argument is therefore *invalid.* Note, however, once again, that to say that an argument is invalid is not to say that it is a bad argument. Under the right circumstances, D would be a perfectly adequate argument. It merely is not one whose conclusion necessarily follows from its premises.

§ 6. Some points of verbal usage. One can intelligibly any of the following: “This argument is valid,” “This inference is invalid,” “This premise is true,” “The conclusion of this argument is false,” “I do not agree with the first premise of this argument.”

One cannot, however, intelligibly say any of the following: “This argument is false,” “This argument is true,” “This premise is valid,” “The conclusion of this argument is valid,” “I agree with this argument.” None of these utterances has any coherent meaning. The notion of validity, as used in logic, applies only to arguments, not to statements; the notions of truth and falsehood apply only to statements, not to arguments; and it is only of what is true or false—hence only of statements, not of arguments—that one can intelligibly say that one agrees or disagrees with it.

§ 7. Validity in general and formal validity. There is no method for deciding, *in every case*, whether an argument is valid or not. One can prove that an argument is *invalid* by finding or conceiving of a state of affairs in which all the premises of the argument are true and yet the conclusion is false. But if one fails to come up with such a state of affairs, that is not sufficient to prove that the argument is valid. The fact that one has failed to conceive of a state of affairs in which the premises are all true and the conclusion false does not entail that no such state of affairs is possible.

There is, however, within the class of valid arguments, a sub-class of arguments such that it is possible to prove that they are valid. These are what are called *formally valid* arguments. A *formally valid* argument is an argument that is valid in virtue of its mere *logical form* (or logical structure). It is possible to define what the logical form of an argument is; however, rather than do that, I will simply present some examples of *valid argument forms*—that is, logical forms that are such that any argument that has one of them is a (formally) valid argument. (Most of these are also illustrated in chapter 6 of *A Rulebook for Arguments*.)[1]

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[1]You may be wondering what would be an example of an argument that is valid, but not formally valid (such an argument is said to be *merely informally valid*). Here is one:

1. Jones is a bachelor.

∴ 2. Therefore, Jones has no wife.

It cannot happen that (1) is true while (2) is not true; thus, the argument is valid. But its validity depends on the meanings of the words involved, rather than on the logical forms of the statements. The validity concerned is therefore not formal but informal validity.

II. SOME VALID ARGUMENT FORMS

§ 8. *Modus ponens*. The term "modus ponens" is a Latin phrase meaning "the manner that affirms." To explain what it means, I need to introduce three more logical terms. A statement of the "If . . . then . . ." form is called a *conditional* statement (or simply a *conditional*); the "if" clause of a conditional statement is called the *antecedent* (literally "that which goes before"); and the "then" clause of a conditional statement is called the *consequent* (literally, "that which follows"). *Modus ponens,* also known as "affirming the antecedent," is a form of argument in which one asserts a conditional statement and the antecedent of the conditional as premises, and infers the consequent of the conditional as the conclusion. An example:

	If P, then Q	If the rent is affordable, the apartment is already taken.
	P	The rent is affordable.
∴	Q	Therefore, the apartment is already taken.

§ 9. *Modus tollens*. The term "modus tollens" is Latin for "the manner that denies," or in plainer terms, "Denying the consequent." This is a form in which one asserts a conditional statement and the denial of the consequent of the conditional as premises, and one infers the negation of the antecedent as the conclusion. Argument B in § 4 was an example of this form:

	If P, then Q	If we have followed the directions, then this is Forest Street.
	Not-Q	This is not Forest Street.
∴	Not-P	Therefore, we have not followed the directions.

§ 10. Hypothetical syllogism. The word "syllogism" derives from the name given by ancient Greek logicians to what we would call simple deductive arguments. (Thus, the two forms of argument already illustrated are also syllogisms.) The term "hypothetical" here means "conditional." The hypothetical syllogism is a form of argument in which a conditional statement is inferred from two other conditional statements, as follows:

	If P, then Q	If the rent is affordable, the apartment is already taken.
	If Q, then R	If the apartment is taken, I can't live there.
∴	If P, then R	Therefore, if the rent is affordable, I can't live there.

§ 11. Inferences with conjunctions. For me to define these forms, I need to introduce two more logical terms. First, a *conjunction*, in the logical sense of the term, is a statement composed of two component statements joined by the word "and." Second, each component statement in a conjunction is called a *conjunct*. Two valid argument forms concerning conjunctions are (i) the form in which one is given a conjunction as a premise and infers one of the conjuncts as a conclusion, and (ii) the form in which one is given two statements as premises, and infers the conjunction of the two statements as the conclusion. Thus we have the following two valid argument forms:

	P and Q	It's cold and it's snowing.
∴	P	Therefore, it's cold.

	P	It's cold.
	Q	It's snowing.
∴	P and Q	Therefore, it's cold and it's snowing.

§ 12. Disjunctive syllogism. Here again, I need to explain two more logical terms. First, a *disjunction* is a statement composed of two statements joined by the word "or." Second, each of the statements combined in a disjunction is called a *disjunct*. A *disjunctive syllogism* is a syllogism in which one premise is a disjunction, one premise is the negation of one of the disjuncts, and the conclusion is the other disjunct of the disjunction. For example:

	P or Q	Either the story is true or Jones is a liar.
	Not-P	The story is not true.
∴	Q	Therefore, Jones is a liar.

It is immaterial to this form whether it is the first or the second disjunct (P or Q) that is negated.

§ 13. Invalid argument forms (i): affirming the consequent. Besides the *valid* argument forms listed above, there are many forms of argument that are sometimes *mistaken* for valid forms. One of these is called *affirming the consequent*, which is a kind of counterfeit of *modus ponens*. It consists in being given a conditional and the consequent of the conditional as premises, and inferring the antecedent of the conditional:

	If P, then Q	If Jones committed the murder, he will look nervous.
	Q	Jones looks nervous.
∴	P	Therefore, Jones committed the murder.

It should be intuitively evident that this argument is invalid: even if both premises are true, Jones may be looking nervous for any number of reasons, while being innocent of any murder. Thus the premises may be true while the conclusion is false, which means that the argument is invalid.

§ 14. Invalid argument forms (ii): denying the antecedent. This form of argument may be regarded as a counterfeit of *modus tollens*. It consists in being given a conditional and the negation of the antecedent of the conditional as premises, and inferring the negation of the consequent:

	If P, then Q	If Jones committed the murder, he will look nervous.
	Not-P	Jones did not commit the murder.
∴	Not-Q	Therefore, Jones will not look nervous.

Once again, it should be intuitively evident that the argument is invalid: the premises may both be true, but the conclusion false, if Jones, while innocent of the murder, looks nervous from other causes.

III. SOME NOTES ON CONDITIONALS

§ 15. "If" versus "only if." There is an important distinction between the use of the expressions "if" and "only if." The distinction may be illustrated by an example. Suppose that there is a health club which allows non-members to use some of its facilities, but not the swimming pool. You, a non-member, ask an attendant if you can use the pool, and the attendant replies with the following statement:

(A) You may use the pool only if you are a member of the club.

You buy a membership, on the assumption that possessing a membership will enable you to use the swimming pool, only to learn that only club members who belong to the family of the founders of the club may use the pool. Was what the attendant said to you untrue? Granted, it may have been misleading (it certainly misled *you*); but it was not false; in fact, it was perfectly true. To say that you may use the pool *only if* you are a member of the club is to say that you *cannot* use the pool if you are *not* a member of the club, or that no one who is not a member of the club may use the pool; it is not to say that anyone who *is* a member of the club *may* use the pool.

On the other hand, suppose that the attendant had said the following:

(B) You may use the pool if and only if you are a member of the club.

To say this is to say that if you are a member of the club then you may use the pool, *and* that if you may use the pool then you are a member of the club. To put the point another way, it says that all and only members of the club may use the pool. When, in our original version of the story, the attendant uttered statement A to you and you took her statement to imply that you would be allowed to use the pool once you got a membership, you were presuming that the attendant meant what is expressed by B (though in fact she meant only and exactly what she said).

A statement of the form "P if and only if Q" is called a *biconditional*. Philosophers and logicians commonly use the expression "iff" to abbreviate "if and only if."

§ 16. Necessary conditions, sufficient conditions, and necessary and sufficient conditions. Suppose that some conditional statement is true—for example, "If the red light is on, a recording is in progress." In that case, the condition expressed by the first clause is said to be a *sufficient condition* for the truth of the second clause: in this example, the red light's being on is a sufficient condition for its being the case that a recording is in progress. Another way of using the expression "sufficient condition" is as follows. Suppose that some "if–then" statement is true in which the two clauses have the same subject; for example, "If you are the President of the United States, then you must have been born in the United States." In that case we can say being the President of the United States is a *sufficient condition* for having been born in the United States.

On the other hand, we may also say that having been born in the United States is a *necessary condition* for being the President of the United States; or, in terms of our first example, that a recording's being in progress is a necessary condition for its being the case that the red light is on.

Finally, given the biconditional "P iff Q," one may say that its being the case that P is a *necessary and sufficient condition* for its being the case that Q (or conversely). For example, a triangle is equilateral if and only if it is equiangular: so being an equilateral triangle is a necessary and sufficient condition of being an equiangular triangle (and conversely).

Philosophers often speak of necessary and sufficient conditions (in the plural) when they are trying to provide an analysis of a concept. For example, if a philosopher ventures to claim that a human subject S knows that P if and only if (i) P is true, (ii) S is sure that P, and (iii) S has the right to be sure that P, then those three conditions are said to be, according to such an analysis, the necessary and sufficient conditions of knowing something.