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PH 101: Problems of Philosophy
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TRUTH CONDITIONS FOR CONDITIONALS

As we have seen, logic assigns truth values to "If P then Q" as follows (table 1):

P	**Q**	**If P then Q**
T	T	T
T	F	F
F	T	T
F	F	T

I have noted that this scheme seems rather counterintuitive. It is clear that, if "P" is true and "Q" false, then "If P then Q" is false (row 2). But is not clear that "If P then Q" is true in the other three cases, as maintained by the truth table. In this document, I offer an informal proof that "If P then Q" is in fact governed by the truth conditions set forth in the table above. The proof consists in arguing that "If P then Q" is logically equivalent to "Not (P and not Q)," then showing that the latter has the truth conditions given in the table above.

1. We are agreed, I take it, that if "If P then Q" is true, then it cannot be the case that "P" is true and "Q" is false. That is to say that the following inference (argument A) is valid:

> "If P then Q" is true.
> Therefore, it is not the case that "P" is true and "Q" is false.

2. Now to say that *"If P then Q" is true* is equivalent to saying "If P then Q." And to say that *"P" is true and "Q" is false* is equivalent to saying "P and not Q." Thus, given that argument A is valid, the following inference (argument B), which is equivalent to it, must also be valid:

> If P then Q.
> Therefore, not (P and not Q).

3. Now let us try to reason in the other direction: we will suppose that "Not (P and not Q)" is true, and see whether "If P then Q" follows from it. To suppose that *"Not (P and not Q)" is true* is equivalent to supposing that "P and not Q" is false. For a conjunction to be false, it must be the case that at least one of its two conjuncts is false. In the present case, this means that, given that the whole statement "P and not Q" is false, at least one of its two conjuncts, "P" and "Not Q," must be false. Note that this does not determine whether either one of those statements is true or false. But it does determine that, if one of them is true, then the other is false.

4. Now suppose that "P" is true. If "P" is true, then, given our supposition that "P and not

Q" is false, it must be the case that the other conjunct of that statement, namely "Not Q," is false. We can sum this up by saying that the following inference is valid:

"Not (P and not Q)" is true.
Therefore, "P and not Q" is false.
Therefore, if "P" is true, then "Not Q" is false.

We can simplify this argument by leaving out the intermediate conclusion, thus (argument C):

"Not (P and not Q)" is true.
Therefore, if "P" is true, then "Not Q" is false.

5. With regard to the first step of argument C, it should be obvious again that to say that *"Not (P and not Q)" is true* is equivalent to saying "Not (P and not Q)." The conclusion of C is more complicated. Obviously, to say that *"P" is true* is equivalent to saying simply *"P."* And to say that *"Not Q" is false* is equivalent to saying that *"Q" is true*, which in turn is equivalent to saying simply "Q." From this it follows that to say that *if "P" is true, then "Not Q" is false* (the conclusion of C) is equivalent to saying *"If P then not Q."* Thus argument C, which we have shown to be valid, is equivalent to the following (argument D), which must also be valid:

Not (P and not Q)
Therefore, if P then Q.

6. Earlier, we showed that the inference from "If P then Q" to "Not (P and not Q)" (argument B) is valid. We have just shown that the inference from "Not (P and not Q)" to "If P then Q" (argument D) is valid. Together, these two proofs show that "If P then Q" is logically equivalent to "Not (P and not Q)": if either is true then the other is also true.

7. Finally, we construct a truth table for "Not (P and not Q)." It will look like this (table 2):

P	**Q**	**Not Q**	**P and not Q**	**Not (P and not Q)**
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

According to this truth table, "Not (P and not Q)" is false just in case "P" is true and "Q" is false; in all other cases, it is true. Now we have proved that "Not (P and not Q)" is logically equivalent to "If P then Q" (step 6). So it follows that "If P then Q" has the same truth conditions: it is false just in case "P" is true and "Q" is false, and true otherwise. These are just the truth conditions assigned to "If P then Q" by logic, as stated in table 1. This proves that "If P then Q" is governed by the truth conditions that logic assigns to it.