Math 221b: Topics in Topology **Spring 2012**
Foliations and contact structures on 3-manifolds

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The course will mainly focus on foliations on 3-manifolds, and some of the many applications. I will start at the beginning and cover the basics, including the construction of taut foliations by gluing from sutured manifolds. Depending on interest of the class, we may also discuss how this interacts with Heegaard-Floer theory and Seiberg-Witten Floer theory. We will discuss also contact structures, but the emphasis will be on foliations.

(1) Basics about foliations (review of Frobenius theorem and its implications) and contact structures.
(2) Some basic theorems and techniques in 3-dimensional topology: loop theorem/Dehn's lemma; incompressible surfaces. Discussion (probably without proofs) of hierarchies.
(3) Existence of foliations and contact structures on 3-manifolds. The fundamental dichotomy in each setting: tight versus overtwisted contact structures, taut versus, um, not taut foliations.
(4) Classical theorems of Novikov, Roussarie, and Thurston about foliations on 3-manifolds.
(5) The Thurston norm on the second homology; compact leaves of taut foliations minimize norm. The unit ball in the Thurston norm is a polytope; fibered faces.
(6) Sutured manifolds; disk decompositions. Gabai's gluing construction for building taut foliations from a disk decomposition.
(7) (If we have the strength.) Sutured manifold hierarchies; Gabai's gluing construction in general; higher depth foliations.
(8) Back to contact structures: local structure (Darboux's theorem). Tight contact structures and genus bounds (Thurston-Bennequin). Other topics: Symplectization; symplectic fillings, Weinstein surgery, open books...

References

(1) *Foliations II*, by A. Candel and L. Conlon. Available from the AMS bookstore.
(2) *A norm for the homology of 3-manifolds*, by W. Thurston, Mem. Amer. Math. Soc. **59** (1986), no. 339.
(3) Introductory Lectures on Contact Geometry by J. Etnyre.
(4) Three-dimensional manifolds; course notes by M. Lackenby.

For background material on foliations I may use some material from Foliations I (A. Candel and L. Conlon) but this material is available elsewhere, particularly in Chapter 6 of *A Comprehensive Introduction to Differential Geometry* Volume 1, by M. Spivak, or chapter 1 of *Foundations of differentiable manifolds and Lie groups* by F. Warner.